

POPULATION GENETICS AND THE HARDY-WEINBERG LAW

The Hardy-Weinberg formulas allow scientists to determine whether evolution has occurred. Any changes in the gene frequencies in the population over time can be detected. The law essentially states that if no evolution is occurring, then an equilibrium of allele frequencies will remain in effect in each succeeding generation of sexually reproducing individuals. In order for equilibrium to remain in effect (i.e. that no evolution is occurring) then the following five conditions must be met:

1. No mutations must occur so that new alleles do not enter the population.
2. No gene flow can occur (i.e. no migration of individuals into, or out of, the population).
3. Random mating must occur (i.e. individuals must pair by chance)
4. The population must be large so that no genetic drift (random chance) can cause the allele frequencies to change.
5. No selection can occur so that certain alleles are not selected for, or against.

Obviously, the Hardy-Weinberg equilibrium cannot exist in real life. Some or all of these types of forces all act on living populations at various times and evolution at some level occurs in all living organisms. The Hardy-Weinberg formulas allow us to detect some allele frequencies that change from generation to generation, thus allowing a simplified method of determining that evolution is occurring. There are two formulas that must be memorized:

$$p^2 + 2pq + q^2 = 1$$
$$p + q = 1$$

<p>p = frequency of the dominant allele in the population q = frequency of the recessive allele in the population</p> <p>p² = percentage of homozygous dominant individuals q² = percentage of homozygous recessive individuals 2pq = percentage of heterozygous individuals</p>

How to solve Hardy-Weinberg Problems:

First, determine allele frequencies. These are the p and q values

1. Find the number of homozygous recessive individuals in the population (your q² value)
2. Find the q value based off the previous information.
3. Use the equation p + q = 1 to determine the p value.

Then, use that information to determine percentages of each genotype in the population

1. Using the p and q values calculated above, calculate the percentages of each type of genotype. Use this equation to do this: $p^2 + 2pq + q^2 = 1$
2. Analyze gene frequencies (e.g., *do these frequencies suggest HW equilibrium? Can you tell if this population is evolving? What do these gene frequencies tell you about the population?*)

Practice Problems

1. You have sampled a population in which you know that the percentage of the homozygous recessive genotype (aa) is 36%. Using that 36%, calculate the following:

A. The frequency of the "aa" genotype.

$$p^2 + 2pq + q^2$$

↓
0.36

B. The frequency of the "a" allele.

$$q^2 = .36$$

$$q = 0.6$$

C. The frequency of the "A" allele.

$$p + q = 1$$

$$p + (0.6) = 1 \quad p = 0.4$$

D. The frequencies of the genotypes "AA" and "Aa."

$$p^2 = (0.4)^2 = 0.16 = AA$$

$$2pq = 2(0.4)(0.6) = 0.48 = Aa$$

E. The frequencies of the two possible phenotypes if "A" is completely dominant over "a."

$$p^2 + 2pq$$

$$(0.16) + (0.48) = 0.64 \text{ dominant; } 0.36 \text{ recessive}$$

2. Sickle-cell anemia is an interesting genetic disease. Normal homozygous individuals (SS) have normal blood cells that are easily infected with the malarial parasite. Thus, many of these individuals become very ill from the parasite and many die. Individuals homozygous for the sickle-cell trait (ss) have red blood cells that readily collapse when deoxygenated. Although malaria cannot grow in these red blood cells, individuals often die because of the genetic defect. However, individuals with the heterozygous condition (Ss) have some sickling of red blood cells, but generally not enough to cause mortality. In addition, malaria cannot survive well within these "partially defective" red blood cells. Thus, heterozygotes tend to survive better than either of the homozygous conditions. If 9% of an African population is born with a severe form of sickle-cell anemia (ss), what percentage of the population will be more resistant to malaria because they are heterozygous (Ss) for the sickle-cell gene?

$$p^2 + 2pq + q^2$$

↓
9%
0.09

$$q^2 = .09$$

$$q = 0.3$$

$$p = 0.7$$

$$Aa = 2pq = 2(0.3)(0.7)$$

$$= .42 = 42\%$$

3. There are 100 students in a class. Ninety-six did well in the course whereas four blew it totally and received a grade of F. Sorry. In the highly unlikely event that these traits are genetic rather than environmental, if these traits involve dominant and recessive alleles, and if the four (4%) represent the frequency of the homozygous recessive condition, please calculate the following:

A. The frequency of the recessive allele.

$$p^2 + 2pq + q^2$$

↓
4%
= .04

$$q^2 = .04$$

$$q = 0.2$$

B. The frequency of the dominant allele.

$$p = 0.8$$

C. The frequency of heterozygous individuals.

$$2pq = 2(0.8)(0.2)$$

$$0.32 = Aa$$

7. After graduation, you and 19 of your closest friends (lets say 10 males and 10 females) charter a plane to go on a round-the-world tour. Unfortunately, you all crash land (safely) on a deserted island. No one finds you and you start a new population totally isolated from the rest of the world. Two of your friends carry (i.e. are heterozygous for) the recessive cystic fibrosis allele (c). Assuming that the frequency of this allele does not change as the population grows, what will be the incidence of cystic fibrosis on your island?

$$p = 0.95$$

$$q = 0.05$$

$$q^2 = (0.05)^2 = .0025$$

$$= 0.25\% \text{ cc}$$

$$n = 20$$

$$18 \text{ CC}$$

$$2 \text{ Cc}$$

$$0 \text{ cc}$$

$$18 + 18 + 2 \text{ "C"} \Rightarrow p$$

$$2 \text{ "c"} \Rightarrow q$$

20 individuals; 40 alleles

$$p = \frac{38}{40} \text{ "C"} = 0.95$$

$$q = \frac{2}{40} \text{ "c"} = 0.05$$

8. Cystic fibrosis is a recessive condition that affects about 1 in 2,500 babies in the Caucasian population of the United States. Please calculate the following.

- A. The frequency of the recessive allele in the population.

$$q = 0.02$$

- B. The frequency of the dominant allele in the population.

$$p = 0.98$$

- C. The percentage of heterozygous individuals (carriers) in the population.

$$2pq = 2(0.02)(0.98) = 0.0392$$

$$.04$$

$$4\% \text{ Cc}$$

$$p^2 + 2pq + q^2$$

$$\downarrow$$

$$.0004 = q^2$$

$$.02 = q$$

$$.98 = p$$

9. In a given population, only the "A" and "B" alleles are present in the ABO-system; there are no individuals with type "O" blood or with O alleles in this particular population. If 200 people have type A blood, 75 have type AB blood, and 25 have type B blood, what are the allelic frequencies of this population (i.e., what are p and q)?

$$\begin{array}{r} \text{AA} = 200 \\ \text{AB} = 75 \\ \text{BB} = 25 \\ \hline \end{array}$$

$$\begin{array}{r} 200 + 200 \\ + 75 \\ \hline 475 \text{ A} \\ \hline 600 \text{ alleles} \end{array} = \begin{array}{l} 0.79 = p \\ 0.21 = q \end{array}$$

10. The ability to taste PTC is due to a single dominate allele "T". You sampled 215 individuals in biology, and determined that 150 could detect the bitter taste of PTC and 65 could not. Calculate all of the potential frequencies.

$$p^2 + 2pq + q^2$$

$$\downarrow$$

$$150 \text{ AA, Aa}$$

$$65 \text{ aa}$$

$$p^2 = (0.45)^2 = 0.20 \text{ AA}$$

$$2pq = 2(0.45)(0.55) = 0.50 \text{ Aa}$$

$$q^2 = (0.55)^2 = 0.30 \text{ aa}$$

$$\frac{65}{210} = 0.30 = q^2$$

$$\begin{array}{l} 0.55 = q \\ 0.45 = p \end{array}$$

4. Within a population of butterflies, the color brown (B) is dominant over the color white (b). And, 40% of all butterflies are white. Given this simple information, which is something that is very likely to be on an exam, calculate the following:

A. The percentage of butterflies in the population that are heterozygous.

$$p^2 + 2pq + q^2$$

\downarrow
 .40

$$q^2 = 0.40$$

$$q = 0.63 \quad p = 0.37$$

$$2pq = 2(0.37)(0.63)$$

$$Aa = 0.47$$

B. The frequency of homozygous dominant individuals.

$$p^2 = (0.37)^2$$

$$AA = 0.14$$

5. A rather large population of Biology instructors have 396 red-sided individuals and 557 tan-sided individuals. Assume that red is totally recessive. Please calculate the following:

A. The allele frequencies of each allele.

$$q = 0.64$$

$$p = 0.36$$

$$p^2 + 2pq + q^2$$

\downarrow
 0.42 = q^2
 0.64 = q

557 tan	red = aa
396 red	
953 individuals	

$red \approx .42 = q^2$

B. The expected genotype frequencies.

$$p^2 = (0.36)^2 = 0.13 \quad AA$$

$$2pq = 2(0.36)(0.64) = 0.46 \quad Aa$$

$$q^2 = (0.64)^2 = 0.41 \quad aa$$

C. The number of heterozygous individuals that you would predict to be in this population.

$$2pq = 0.46$$

$$0.46 \left(\underset{\text{total}}{953} \right) = 438 \quad Aa$$

D. The expected phenotype frequencies.

$$AA + Aa$$

$$0.13 + 0.46 = 0.59 \quad \text{dominant (tan)}$$

$$0.41 \quad \text{recessive (red)}$$

E. Conditions happen to be really good this year for breeding and next year there are 1,245 young "potential" Biology instructors. Assuming that all of the Hardy-Weinberg conditions are met, how many of these would you expect to be red-sided and how many tan-sided?

$$0.59 (1,245) = 734.55 = 735 \quad \text{tan}$$

$$0.41 (1,245) = 510.45 = 510 \quad \text{red}$$

6. A very large population of randomly-mating laboratory mice contains 35% white mice. White coloring is caused by the double recessive genotype, "aa". Calculate allelic and genotypic frequencies for this population.

$$p^2 + 2pq + q^2$$

\downarrow
 \downarrow
 0.35 = q^2
 .6 = q
 .4 = p

$$p^2 = (0.4)^2 = 0.16 \quad AA$$

$$2pq = 2(0.4)(0.6) = 0.48 \quad Aa$$

$$q^2 = (0.6)^2 = 0.36 \quad aa$$