

TPCT's

College of Engineering, Osmanabad

Laboratory Manual

Theory of Machine-I

For

Third Year Students

Manual Prepared by

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TPCT's

College of Engineering

Solapur Road, Osmanabad

Department of Mechanical engineering

Vision of the Department:

To impart strong foundation in Mechanical Engineering Fundamentals, so that students will be competent professionals to meet the global challenges

Mission of the Department:

To promote scientific & educational activities for facing problems of global competition and prepare engineering students for successful carriers.

College of Engineering

Technical Document

This technical document is a series of Laboratory manuals of Mechanical engineering Department and is a certified document of College of engineering, Osmanabad. The care has been taken to make the document error-free. But still if any error is found, kindly bring it to the notice of subject teacher and HOD.

Recommended by,

HOD

Approved by,

Principal

FOREWORD

It is my great pleasure to present this laboratory manual for third year engineering students for the subject of Theory of Machine-I to understand the basic concepts of Link, Mechanism, Velocity and Acceleration analysis, Cam, brakes and Dynamometer

This is a core subject to study and understands the Mechanisms of machine. This lab manual provides a platform to the students for understanding different mechanisms of machines and their analysis.

This practical background will help students to select the different mechanism to design the machines.

H.O.D

MECH Dept

LABORATORY MANUAL CONTENTS

This manual is intended for the Third Year students of MECH branches in the subject of Theory of Machines-I. This manual typically contains practical/ Lab Sessions related to Theory of Machines-I covering various aspects related to the subject for enhanced understanding.

Students are advised to thoroughly go through this manual rather than only topics mentioned in the syllabus as practical aspects are the key to understanding and conceptual visualization of theoretical aspects covered in the books.

SUBJECT INDEX:

1. Do's & Don'ts in Laboratory.

2. Lab Exercises

1. Study of Kinematics and Definition
2. Solution of minimum two problems each on determination of velocity using relative velocity and instantaneous center method.
3. Solution of minimum two problems each on relative acceleration method, involving Coriolis acceleration and one each on short cut methods (Klien Construction, Ritterhaus Construction, Modified kien Construction)
4. Solution of minimum three (including graphical and analytical) problems on topic no- 4
5. Solution of three problem on topic-5
6. Study of Brakes
7. Study of Dynamometer.

3. Quiz

4. Conduction of viva voce examination

5. Evaluation & marking scheme

Dos and Don'ts in Laboratory :-

1. Work deliberately and carefully.
2. Keep your work area clean.
3. Students must wear college uniform and carry their college ID.
4. Students should have separate note book for practical.
5. Students should have their own pencil, eraser, scale, along with pen and lab note book.
6. Students should re-write all experiments on standard journal paper.
7. Students found absent for lab will not be allowed to attend next practical without permission of C.C. /H.O.D.

Handle the equipment /models carefully..

Instruction for Laboratory Teachers:-

1. Submission related to whatever lab work has been completed should be done during the next lab session.
2. Students should be instructed to switch on the power supply after getting the checked by the lab assistant / teacher. After the experiment is over, the students must hand over the model of equipment to the lab assistant/teacher.
3. The promptness of submission should be encouraged by way of marking and evaluation patterns that will benefit the sincere students.

Experiment No: 1

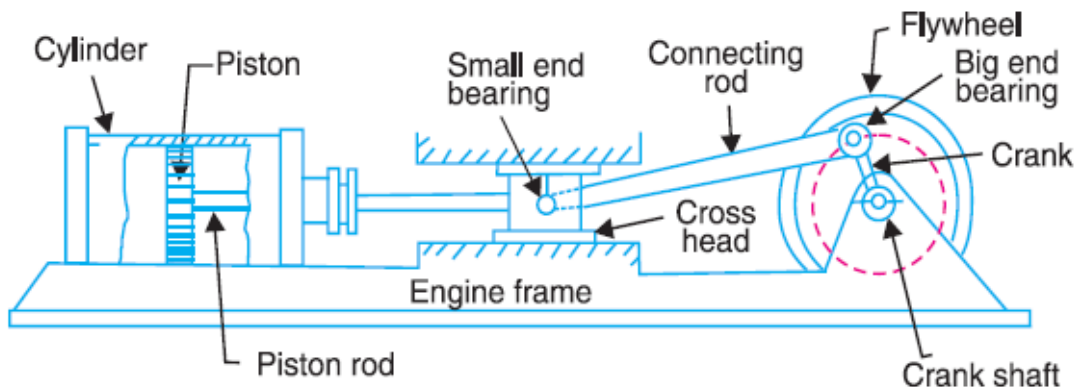
Study of Kinematics and definitions

Introduction:

Machine is a device which receives energy and transforms it into some useful work. A machine consists of a number of parts or bodies. we shall study the mechanisms of the various parts or bodies from which the machine is assembled. This is done by making one of the parts as fixed, and the relative motion of other parts is determined with respect to the fixed part.

Kinematic Link or Element:

Each part of a machine, which moves relative to some other part, is known as a kinematic link (or simply link) or element. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. For example, in a reciprocating steam engine, as shown in Fig. piston, piston rod and crosshead constitute one link ; connecting rod with big and small end bearings constitute a second link ; crank, crank shaft and flywheel a third link and the cylinder, engine frame and main bearings a fourth link.



A link or element needs not to be a rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus a link should have the following two characteristics:

1. It should have relative motion, and
2. It must be a resistant body.

Types of Links

In order to transmit motion, the driver and the follower may be connected by the following three types of links :

1. Rigid link. A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

2. Flexible link. A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

3. Fluid link. A fluid link is one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Structure:

It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

Difference between a Machine and a Structure

The following differences between a machine and a structure are important from the subject point of view:

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.
2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.
3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

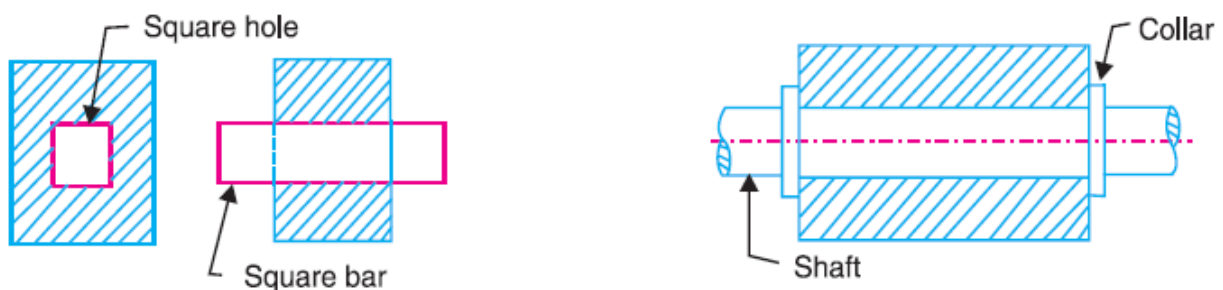
Kinematic Pair

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

Types of Constrained Motions

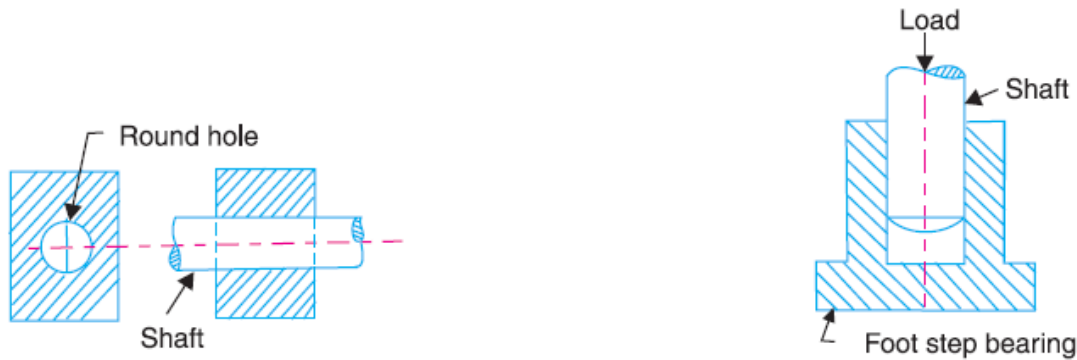
Following are the three types of constrained motions:

1. **Completely constrained motion.** When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion. For example, the piston and cylinder (in a steam engine) form a pair and the motion of the piston is limited to a definite direction (i.e. it will only reciprocate) relative to the cylinder irrespective of the direction of motion of the crank, as shown in Fig



2. **Incompletely constrained motion.** When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of impressed force may alter the direction of relative motion between the pair. A circular bar or shaft in a circular hole, as shown in Fig. is an example of an incompletely

constrained motion as it may either rotate or slide in a hole. These both motions have no relationship with the other



3. Successfully constrained motion.

When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion. Consider a shaft in a foot-step bearing as shown in Fig.. The shaft may rotate in a bearing or it may move upwards. This is a case of incompletely constrained motion. But if the load is placed on the shaft to prevent axial upward movement of the shaft, then the motion of the pair is said to be successfully constrained motion. The motion of an I.C. engine valve (these are kept on their seat by a spring) and the piston reciprocating inside an engine cylinder are also the examples of successfully constrained motion.

Classification of Kinematic Pairs

The kinematic pairs may be classified according to the following considerations:

1. **According to the type of relative motion between the elements.** The kinematic pairs according to type of relative motion between the elements may be classified as discussed below:

(a) **Sliding pair.** When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

(b) **Turning pair.** When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

(c) **Rolling pair.** When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

(d) **Screw pair.** When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut, and bolt with a nut are examples of a screw pair.

(e) **Spherical pair.** When the two elements of a pair are connected in such a way that one element (with spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

2. According to the type of contact between the elements. The kinematic pairs according to the type of contact between the elements may be classified as discussed below :

(a) Lower pair. When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair. It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

(b) Higher pair. When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair. A pair of friction discs, toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

3. According to the type of closure. The kinematic pairs according to the type of closure between the elements may be classified as discussed below :

(a) Self closed pair. When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self closed pair. The lower pairs are self closed pair.

(b) Force - closed pair. When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

Inversion of Mechanism

We have already discussed that when one of links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

It may be noted that the relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically

Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important from the subject point of view:

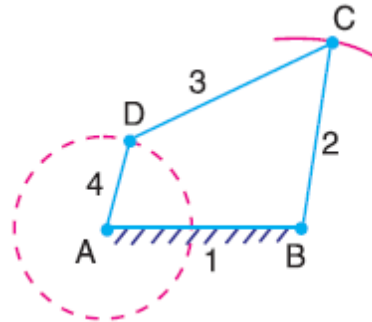
1. Four bar chain or quadric cyclic chain,
2. Single slider crank chain, and
3. Double slider crank chain.

These kinematic chains are discussed, in detail, in the following articles.

Four Bar Chain or Quadric Cycle Chain

We have already discussed that the kinematic chain is a combination of four or more kinematic pairs, such that the relative motion between the links or elements is completely constrained. The simplest and the basic kinematic chain is a four bar chain or quadric cycle chain, as shown in Fig. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths. According to Grashof's law for a four bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links. A very important consideration in designing a mechanism is to ensure that the input crank makes a complete revolution relative to the other links. The mechanism in which no link makes a complete

revolution will not be useful. In a four bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof's law. Such a link is known as crank or driver. In Fig. AD (link 4) is a crank. The link BC (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link CD (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link AB (link 1) is known as frame of the mechanism.

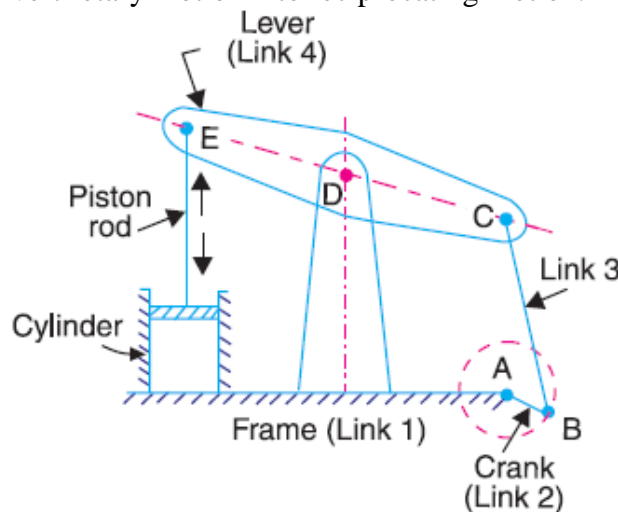


Inversions of Four Bar Chain

Though there are many inversions of the four bar chain, yet the following are important from the subject point of view :

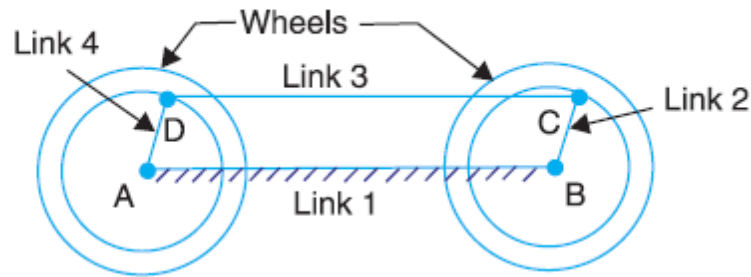
1. Beam engine (crank and lever mechanism)

. A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links, is shown in Fig. In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.



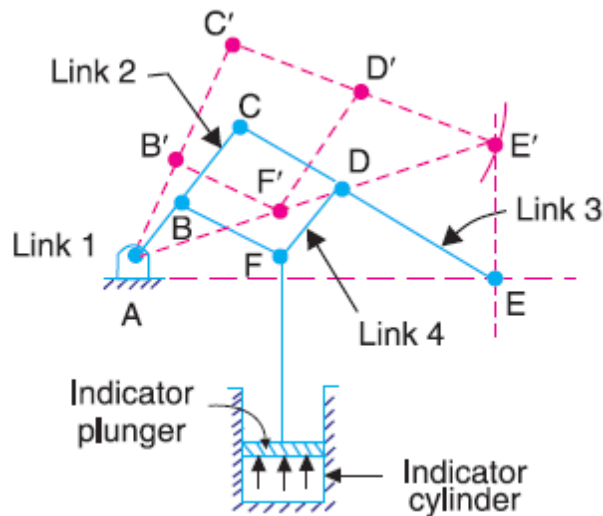
2. Coupling rod of a locomotive (Double crank mechanism).

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in Fig. In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them. This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.



3. Watt's indicator mechanism (Double lever mechanism).

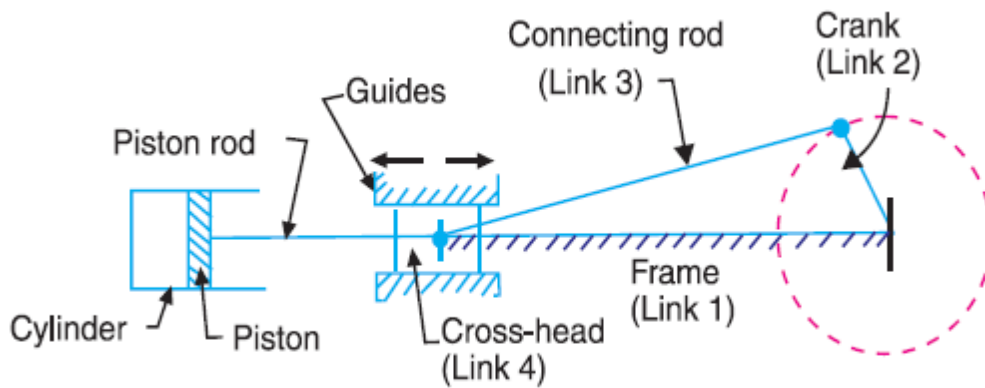
A Watt's indicator mechanism (also known as Watt's straight line mechanism or double lever mechanism) which consists of four links, is shown in Fig. The four links are : fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger. On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.



The initial position of the mechanism is shown in Fig. by full lines whereas the dotted lines show the position of the mechanism when the gas or steam pressure acts on the indicator plunger.

Single Slider Crank Chain

A single slider crank chain is a modification of the basic four bar chain. It consists of one sliding pair and three turning pairs. It is, usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa. In a single slider crank chain, as shown in Fig, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair.



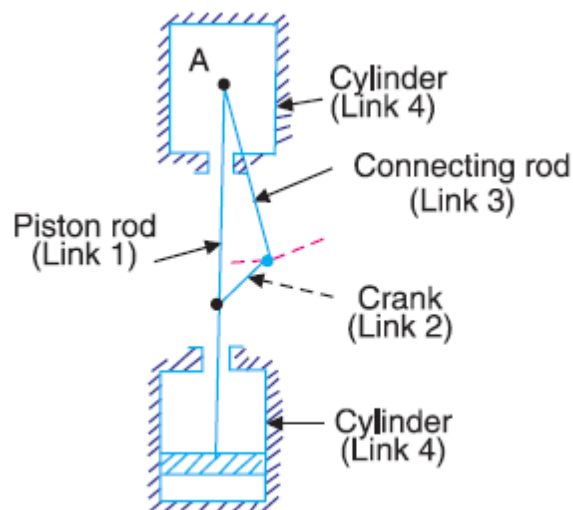
The link 1 corresponds to the frame of the engine, which is fixed. The link 2 corresponds to the crank ; link 3 corresponds to the connecting rod and link 4 corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

Inversions of Single Slider Crank Chain

We have seen in the previous article that a single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious, that four inversions of a single slider crank chain are possible. These inversions are found in the following mechanisms.

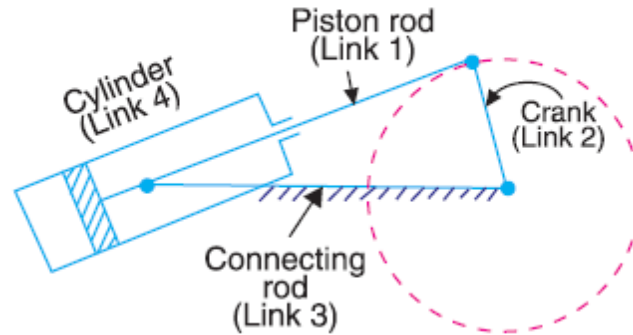
1. Pendulum pump or Bull engine.

In this mechanism, the inversion is obtained by fixing the cylinder or link 4 (i.e. sliding pair), as shown in Fig. In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates. The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1, as shown in Fig.



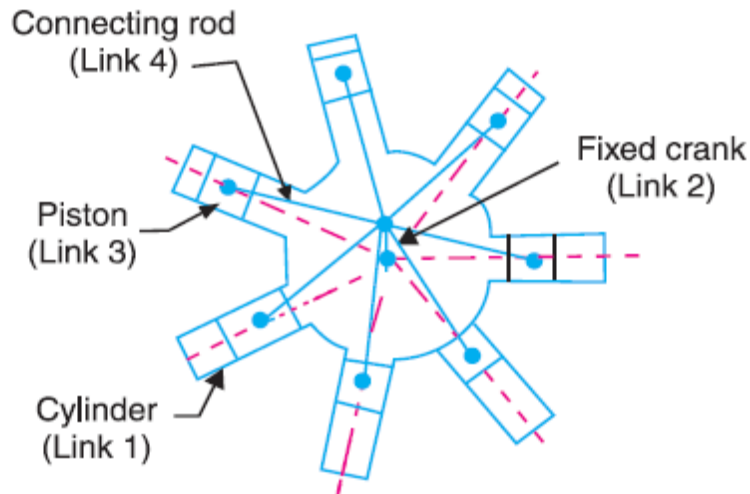
2. Oscillating cylinder engine.

The arrangement of oscillating cylinder engine mechanism, as shown in Fig, is used to convert reciprocating motion into rotary motion. In this mechanism, the link 3 forming the turning pair is fixed. The link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.



3. Rotary internal combustion engine

Sometimes back, rotary internal combustion engines were used in aviation. But now-a-days gas turbines are used in its place. It consists of seven cylinders in one plane and all revolves about fixed centre D, as shown in Fig. While the crank (link 2) is fixed. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

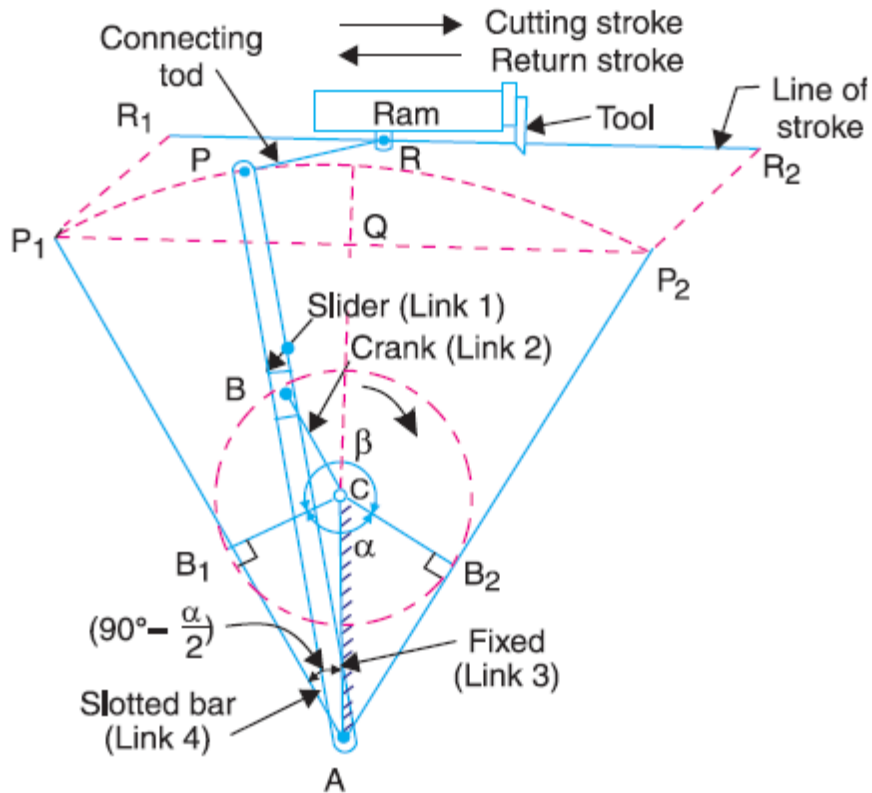


4. Crank and slotted lever quick return motion mechanism.

This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed, as shown in Fig. The link 3 corresponds to the connecting rod of a reciprocating steam engine. The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool

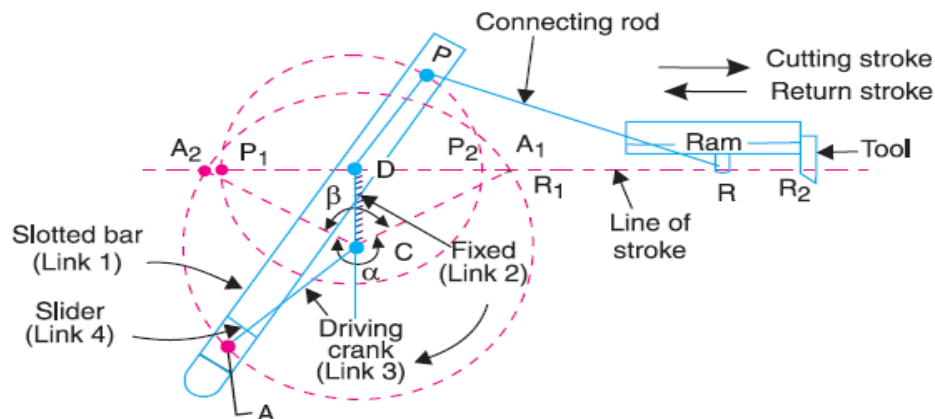
and reciprocates along the line of stroke R1R2. The line of stroke of the ram (i.e. R1R2) is perpendicular to AC produced.



In the extreme positions, AP1 and AP2 are tangential to the circle and the cutting tool is at the end of the stroke. The forward or cutting stroke occurs when the crank rotates from the position CB1 to CB2 (or through an angle) in the clockwise direction. The return stroke occurs when the crank rotates from the position CB2 to CB1 (or through angle) in the clockwise direction. Since the crank has uniform angular speed.

5. Whitworth quick return motion mechanism.

This mechanism is mostly used in shaping and slotting machines. In this mechanism, the link CD (link 2) forming the turning pair is fixed, as shown in Fig. The link 2 corresponds to a crank in a reciprocating steam engine.



The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD

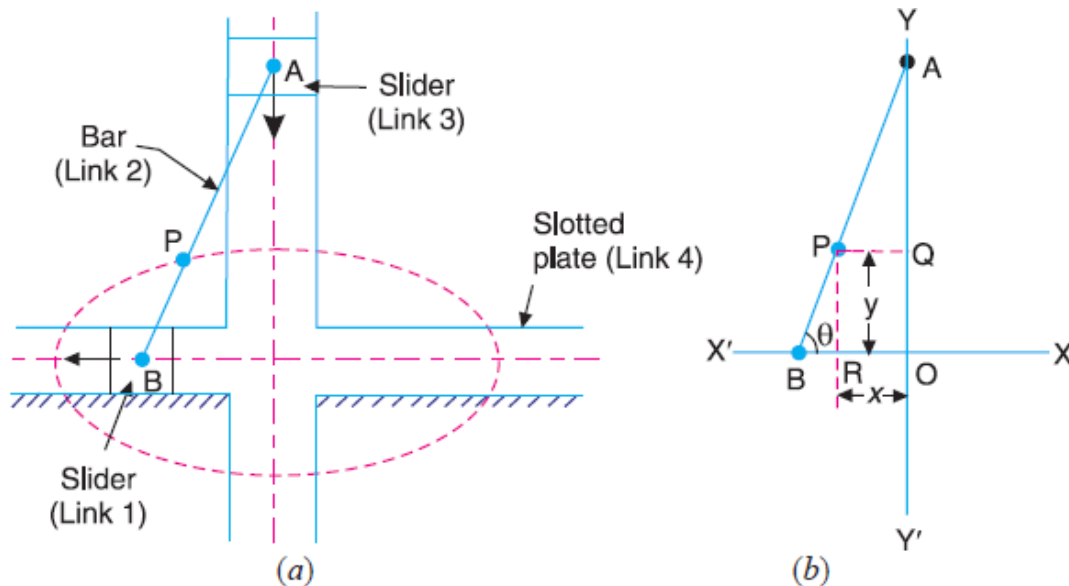
Double Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as double slider crank chain

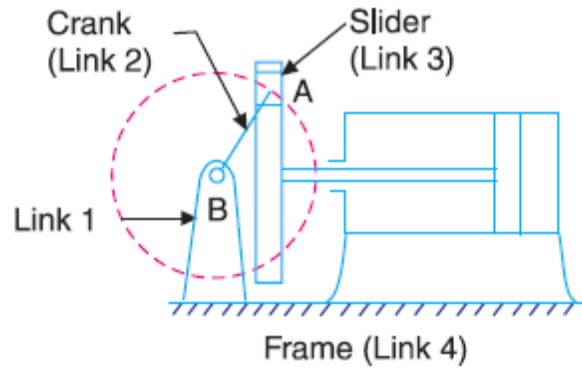
Inversions of Double Slider Crank Chain

The following three inversions of a double slider crank chain are important from the subject point of view:

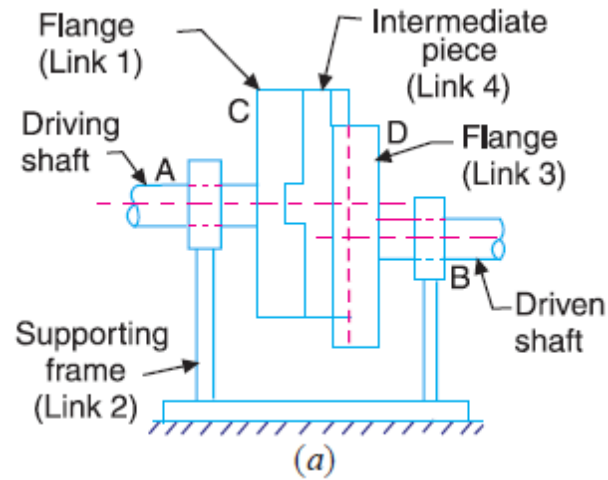
1. Elliptical trammels. It is an instrument used for drawing ellipses. This inversion is obtained by fixing the slotted plate (link 4), as shown in Fig. The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. The link 1 and link 3, are known as sliders and form sliding pairs with link 4. The link AB (link 2) is a bar which forms turning pair with links 1 and 3. When the links 1 and 3 slide along their respective grooves, any point on the link 2 such as P traces out an ellipse on the surface of link 4, as shown in Fig. (a). A little consideration will show that AP and BP are the semi-major axis and semi-minor axis of the ellipse respectively.



2. Scotch yoke mechanism. This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig. link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.



3. **Oldham's coupling.** An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is obtained by fixing the link 2, as shown in Fig. (a). The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.



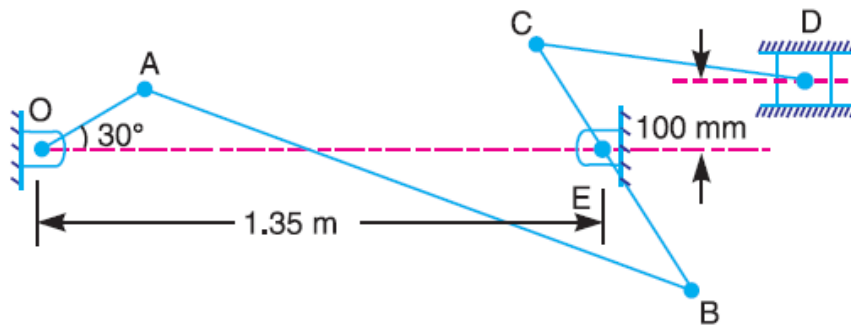
Experiment No: 2

Solution of minimum two problems each on velocity determination by Relative Velocity. Method and Instantaneous Centre Method

Example 1. A mechanism, as shown in Fig. has the following dimensions:

OA = 200 mm; AB = 1.5 m; BC = 600 mm; CD = 500 mm and BE = 400 mm. Locate all the instantaneous centres.

If crank OA rotates uniformly at 120 r.p.m. clockwise, find 1. the velocity of B, C and D, 2. the angular velocity of the links AB, BC and CD.



Solution:

Given : $N_{OA} = 120$ r.p.m. or $\omega_{OA} = 2\pi \times 120/60 = 12.57$ rad/s

Since the length of crank OA = 200 mm = 0.2 m, therefore linear velocity of crank OA,

$$V_{OA} = V_A = \omega_{OA} \times OA = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

Location of instantaneous centres

The instantaneous centres are located as discussed below:

1. Since the mechanism consists of six links (i.e. $n = 6$), therefore the number of instantaneous centres,

$$N = \frac{n(n-1)}{2} = 15 \text{ as } n=6$$

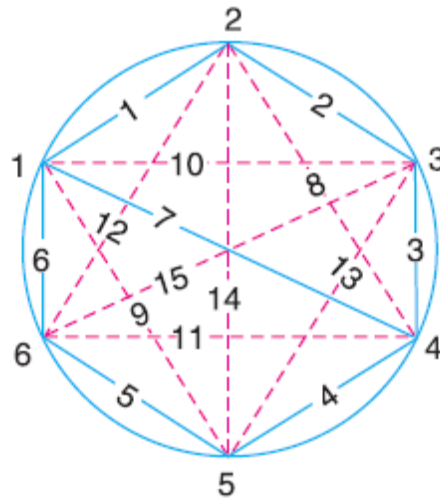
link	1	2	3	4	5	6
I.C.	12	23	34	45	56	
	13	24	35	46		
	14	25	36			
	15	26				
	16					

2. Make a list of all the instantaneous centres in a mechanism. Since the mechanism has 15 instantaneous centres, therefore these centres are listed in the following book keeping table.

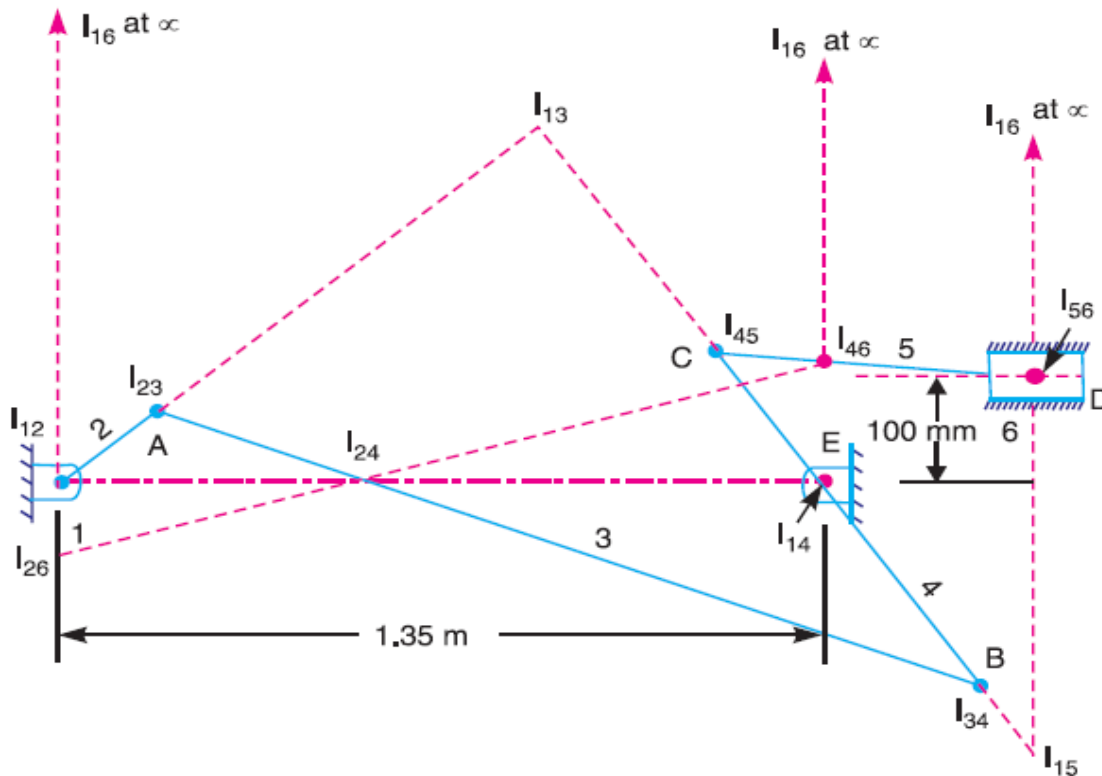
3. Locate the fixed and permanent instantaneous centres by inspection. These centres are I12 I23, I34, I45, I56, I16 and I14 as shown in Fig. 6.16.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. Draw a circle and mark points equal to the number of links such as 1, 2, 3, 4, 5 and 6 as shown in Fig. 6.17. Join the points 12, 23, 34, 45, 56, 61 and 14 to indicate the centres I12, I23, I34, I45, I56, I16 and I14 respectively.

5. Join point 2 to 4 by a dotted line to form the triangles 1 2 4 and 2 3 4. The side 2 4, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{24} lies on the intersection of $I_{12} I_{14}$ and $I_{23} I_{34}$ produced if necessary. Thus centre I_{24} is located. Mark number 8 on the dotted line 2 4 (because seven centres have already been located).



6. Now join point 1 to 5 by a dotted line to form the triangles 1 4 5 and 1 5 6. The side 1 5, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I_{15} lies on the intersection of $I_{14} I_{45}$ and $I_{56} I_{16}$ produced if necessary. Thus centre I_{15} is located. Mark number 9 on the dotted line 1 5.



7. Join point 1 to 3 by a dotted line to form the triangles 1 2 3 and 1 3 4. The side 1 3, common to both triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I13 lies on the intersection I12 I23 and I34 I14 produced if necessary. Thus centre I13 is located. Mark number 10 on the dotted line 1 3.

8. Join point 4 to 6 by a dotted line to form the triangles 4 5 6 and 1 4 6. The side 4 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I46 lies on the intersection of I45 I56 and I14 I16. Thus centre I46 is located. Mark number 11 on the dotted line 4 6.

9. Join point 2 to 6 by a dotted line to form the triangles 1 2 6 and 2 4 6. The side 2 6, common to both triangles, is responsible for completing the two triangles. Therefore, centre I26 lies on the intersection of lines joining the points I12 I16 and I24 I46. Thus centre I26 is located. Mark number 12 on the dotted line 2 6.

10. In the similar way the thirteenth, fourteenth and fifteenth instantaneous centre (i.e. I35, I25 and I36) may be located by joining the point 3 to 5, 2 to 5 and 3 to 6 respectively.

By measurement, we find that

I13 A = 840 mm = 0.84 m ; I13 B = 1070 mm = 1.07 m ; I14 B = 400 mm = 0.4 m ;

I14 C = 200 mm = 0.2 m ; I15 C = 740 mm = 0.74 m ; I15 D = 500 mm = 0.5 m

1. Velocity of points B, C and D

Let V_B , V_C and V_D = Velocity of the points B, C and D respectively.

Example: 2

Fig. shows a sewing needle bar mechanism O1ABO2CD wherein the different dimensions are as follows: Crank O1A = 16 mm; $\beta = 45^\circ$; Vertical distance between O1 and O2 = 40 mm; Horizontal distance between O1 and O2 = 13 mm; O2 B = 23 mm; AB = 35 mm; Angle O2 BC = 90° ; BC = 16 mm; CD = 40 mm. D lies vertically below O1. Find the velocity of needle at D for the given configuration. The crank O1A rotates at 400 r.p.m.

Solution. Given : $N_{O1A} = 400$ r.p.m or $\omega_{O1A} = 2\pi \times 400/60 = 41.9$ rad/s ; $O1 A = 16$ mm = 0.016 m

We know that linear velocity of the crank O1A, $v_{O1A} = v_A = \omega_{O1A} \times O1A = 41.9 \times 0.016 = 0.67$ m/s Now let us locate the required instantaneous centres as discussed below :

1. Since the mechanism consists of six links (i.e. $n = 6$), therefore number of instantaneous centers

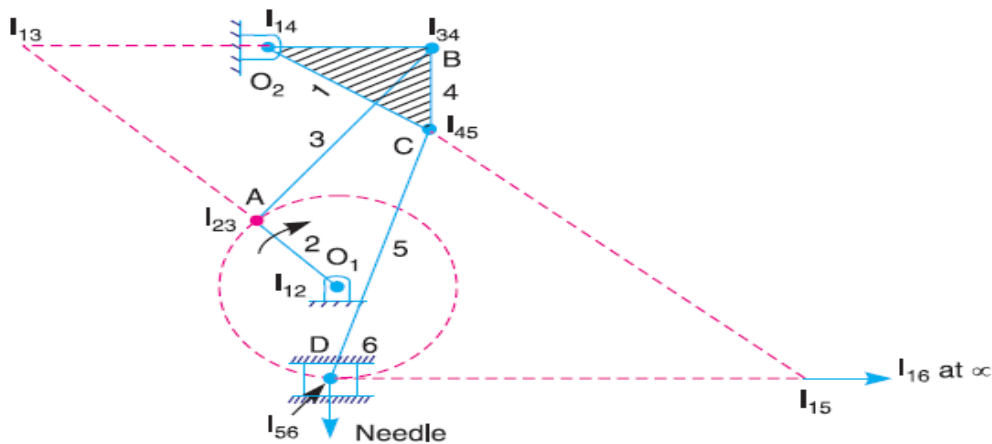
$$N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$$

2. Since the mechanism has 15 instantaneous centres, therefore these centres may be listed in the book keeping table, as discussed in Example

3. Locate the fixed and permanent instantaneous centres by inspections. These centres are I12, I23, I34, I45, I56, I16 and I14, as shown in fig.

4. Locate the remaining neither fixed nor permanent instantaneous centres by Aronhold Kennedy's theorem. This is done by circle diagram as shown in Fig. Mark six points on the

circle (i.e. equal to the number of links in a mechanism) and join 1 to 2, 2 to 3, 3 to 4, 4 to 5, 5 to 6, 6 to 1 and 1 to 4 to indicate the fixed and permanent instantaneous centres i.e. I₁₂, I₂₃, I₃₄, I₄₅, I₅₆, I₁₆ and I₁₄ respectively.



5. Join 1 to 3 by a dotted line to form two triangles 1 2 3 and 1 3 4. The side 1 3, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I₁₃ lies on the intersection of I₁₂ I₂₃ and I₁₄ I₃₄ produced if necessary. Thus centre I₁₃ is located. Mark number 8 (because seven centres have already been located) on the dotted line 1 3.

6. Join 1 to 5 by a dotted line to form two triangles 1 5 6 and 1 4 5. The side 1 5, common to both the triangles, is responsible for completing the two triangles. Therefore the instantaneous centre I₁₅ lies on the intersection of I₁₆ I₅₆ and I₁₄ I₄₅ produced if necessary. Thus centre I₁₅ is located. Mark number 9 on the dotted line 1 5.

Note: For the given example, we do not require other instantaneous centres.

By measurement, we find that

$$I_{13}A = 41 \text{ mm} = 0.041 \text{ m} ; I_{13}B = 50 \text{ mm} = 0.05 \text{ m} ; I_{14}B = 23 \text{ mm} = 0.023 \text{ m} ;$$

$$I_{14}C = 28 \text{ mm} = 0.028 \text{ m} ; I_{15}C = 65 \text{ mm} = 0.065 \text{ m} ; I_{15}D = 62 \text{ mm} = 0.062 \text{ m}$$

$$v_B = \text{Velocity of point } B,$$

$$v_C = \text{Velocity of point } C, \text{ and}$$

$$v_D = \text{Velocity of the needle at } D.$$

We know that $\frac{v_A}{I_{13}A} = \frac{v_B}{I_{13}B}$... (Considering centre I₁₃)

$$\therefore v_B = \frac{v_A}{I_{13}A} \times I_{13}B = \frac{0.67}{0.041} \times 0.05 = 0.817 \text{ m/s}$$

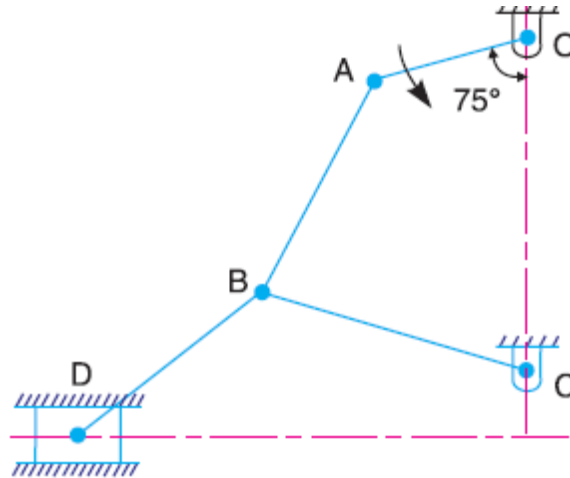
and $\frac{v_B}{I_{14}B} = \frac{v_C}{I_{14}C}$... (Considering centre I₁₄)

$$\therefore v_C = \frac{v_B}{I_{14}B} \times I_{14}C = \frac{0.817}{0.023} \times 0.028 = 0.995 \text{ m/s}$$

Similarly, $\frac{v_C}{I_{15}C} = \frac{v_D}{I_{15}D}$... (Considering centre I₁₅)

$$\therefore v_D = \frac{v_C}{I_{15}C} \times I_{15}D = \frac{0.995}{0.065} \times 0.062 = 0.95 \text{ m/s} \text{ Ans.}$$

Example:3 In Fig., the angular velocity of the crank OA is 600 r.p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are : OA = 28 mm ; AB = 44 mm ; BC 49 mm ; and BD = 46 mm. The centre distance between the centres of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



Solution. Given: $N_{AO} = 600$ r.p.m. or $\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s

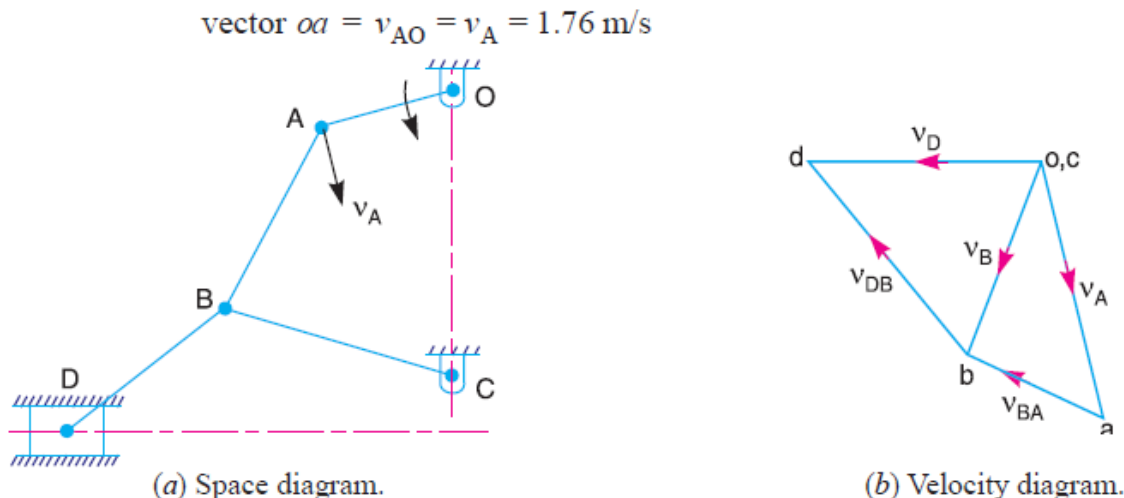
Since OA = 28 mm = 0.028 m, therefore velocity of A with respect to O or velocity of A (because O is a fixed point)

$$v_{AO} = v_A = \omega_{AO} \times OA = 62.84 \times 0.028 = 1.76 \text{ m/s}$$

Linear velocity of the slider D

First of all draw the space diagram, to some suitable scale, as shown in Fig. (a) Now the velocity diagram, as shown in Fig. (b), is drawn as discussed below:

1. Since the points O and C are fixed, therefore these points are marked as one point, in the velocity diagram. Now from point o, draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O or simply velocity of A such that.



- From point a, draw vector ab perpendicular to AB to represent the velocity of B with respect to A (i.e. v_{BA}) and from point c, draw vector cb perpendicular to CB to represent the velocity of B with respect to C or simply velocity of B (i.e. v_{BC} or v_B). The vectors ab and cb intersect at b.
- From point b, draw vector bd perpendicular to BD to represent the velocity of D with respect to B (i.e. v_{DB}) and from point o, draw vector od parallel to the path of motion of the slider D which is horizontal, to represent the velocity of D (i.e. v_D). The vectors bd and od intersect at d. By measurement, we find that velocity of the slider D,

$$V_D = \text{vector } od = 1.6 \text{ m/s}$$

Angular velocity of the link BD

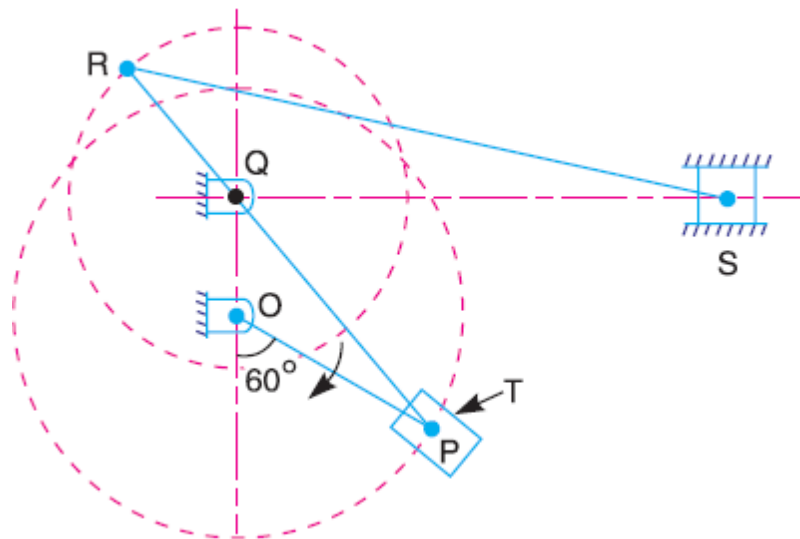
By measurement from velocity diagram, we find that velocity of D with respect to B,

$$v_{DB} = \text{vector } bd = 1.7 \text{ m/s}$$

Since the length of link $BD = 46 \text{ mm} = 0.046 \text{ m}$, therefore angular velocity of the link BD,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96 \text{ rad/s (Clockwise about B) Ans.}$$

Example 4. Fig. shows the structure of Whitworth quick return mechanism used in reciprocating machine tools. The various dimensions of the tool are as follows : $OQ = 100 \text{ mm}$; $OP = 200 \text{ mm}$, $RQ = 150 \text{ mm}$ and $RS = 500 \text{ mm}$. The crank OP makes an angle of 60° with the vertical. Determine the velocity of the slider S (cutting tool) when the crank rotates at 120 r.p.m. clockwise. Find also the angular velocity of the link RS and the velocity of the sliding block T on the slotted lever QT.



Solution. Given : $N_{PO} = 120 \text{ r.p.m.}$ or $\omega_{PO} = 2\pi \times 120/60 = 12.57 \text{ rad/s}$

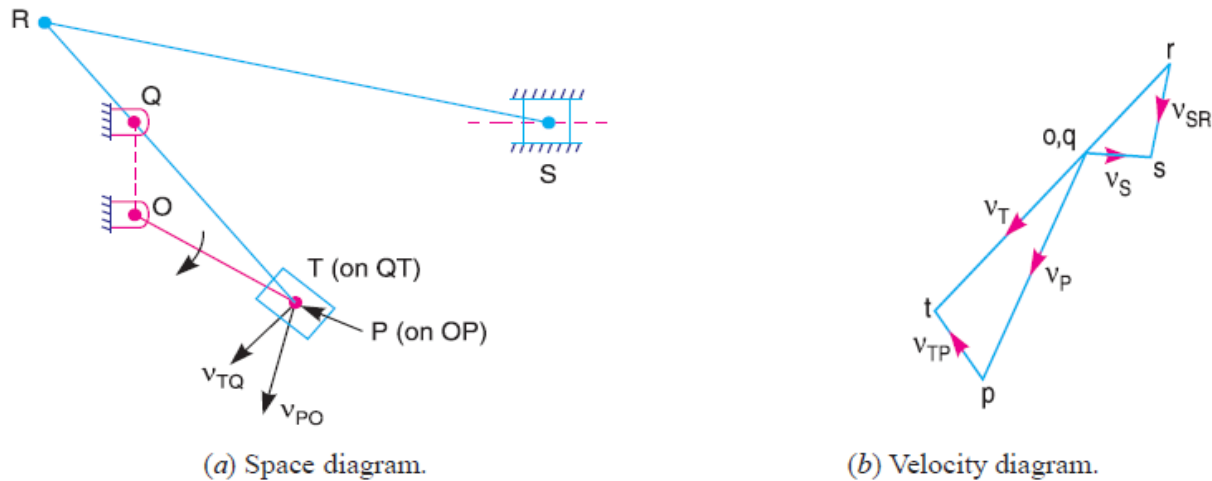
Since the crank $OP = 200 \text{ mm} = 0.2 \text{ m}$, therefore velocity of P with respect to O or velocity of P (because O is a fixed point)

$$v_{PO} = v_P = \omega_{PO} \times OP = 12.57 \times 0.2 = 2.514 \text{ m/s}$$

Velocity of slider S (cutting tool)

First of all draw the space diagram, to some suitable scale, as shown in Fig. (a). Now the velocity diagram, as shown in Fig. (b) is drawn as discussed below:

1. Since O and Q are fixed points, therefore they are taken as one point in the velocity diagram. From point o, draw vector op perpendicular to OP, to some suitable scale, to represent the velocity of P with respect to O or simply velocity of P, such that.



2. From point q, draw vector qt perpendicular to QT to represent the velocity of T with respect to Q or simply velocity of T (i.e. v_{TQ} or v_T) and from point p draw vector pt parallel to the path of motion of T (which is parallel to TQ) to represent the velocity of T with respect to P (i.e. v_{TP}). The vectors qt and pt intersect at t.

3. Since the point R lies on the link TQ produced, therefore divide the vector tq at r in the same ratio as R divides TQ, in the space diagram. In other words

$$qr/qt = QR/QT$$

The vector qr represents the velocity of R with respect to Q or velocity of R (i.e. v_{RQ} or v_R)

4. From point r, draw vector rs perpendicular to RS to represent the velocity of S with respect to R and from point o draw vector or parallel to the path of motion of S (which is parallel to QS) to represent the velocity of S (i.e. v_S). The vectors rs and os intersect at s. By measurement, we find that velocity of the slider S (cutting tool)

$$v_S = \text{vector } os = 0.8 \text{ m/s}$$

Angular velocity of link RS From the velocity diagram, we find that the linear velocity of the link RS

$$v_{SR} = \text{vector } rs = 0.96 \text{ m/s}$$

Since the length of link RS = 500 mm = 0.5 m, therefore angular velocity of link RS

$$v_{SR} = \text{vector } rs = 0.96 \text{ m/s}$$

Since the length of link RS = 500 mm = 0.5 m, therefore angular velocity of link RS,

$$\omega_{RS} = \frac{v_{SR}}{RS} = \frac{0.96}{0.5} = 0.92 \text{ rad/s (Clockwise about R) Ans.}$$

Velocity of the sliding block T on the slotted lever QT

Since the block T moves on the slotted lever with respect to P, therefore velocity of the sliding block T on the slotted lever QT,

$$v_{TP} = \text{vector } pt = 0.85 \text{ m/s Ans.} \quad \dots \text{ (By measurement)}$$

Experiment No: 3

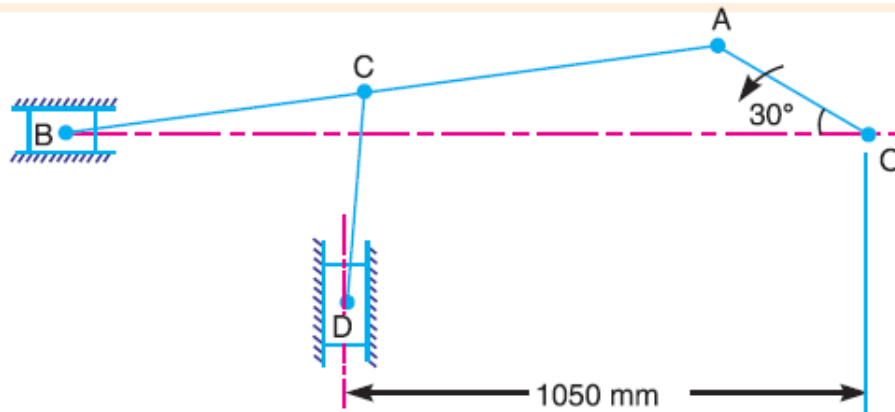
Solution of minimum two problems each on relative acceleration Method, involving Coriolis Acceleration and one each on short cut Methods

Example 1. In the mechanism, as shown in , the crank OA rotates at 20 r.p.m. anticlockwise and gives motion to the sliding blocks B and D. The dimensions of the various links are OA = 300 mm; AB = 1200 mm; BC = 450 mm and CD = 450 mm. For the given configuration, determine : 1. velocities of sliding at B and D, 2. Angular velocity of CD, 3. linear acceleration of D, and 4. angular acceleration of CD.

Solution:

Given : $N_{AO} = 20$ r.p.m. or $\omega_{AO} = 2\pi \times 20/60 = 2.1$ rad/s ; OA = 300 mm = 0.3 m ;
AB = 1200 mm = 1.2 m ; BC = CD = 450 mm = 0.45 m

We know that linear velocity of A with respect to O or velocity of A,
 $V_{AO} = V_A = \omega_{AO} \times OA = 2.1 \times 0.3 = 0.63$ m/s ...(Perpendicular to OA)



1. Velocities of sliding at B and D

First of all, draw the space diagram, to some suitable scale. Now the velocity diagram is drawn as discussed below:

1. Draw vector oa perpendicular to OA, to some suitable scale, to represent the velocity of A with respect to O (or simply velocity of A), such that

vector $oa = v_{AO} = v_A = 0.63$ m/s

2. From point a , draw vector ab perpendicular to AB to represent the velocity of B with respect to A (i.e. v_{BA}) and from point o draw vector ob parallel to path of motion B (which is along BO) to represent the velocity of B with respect to O (or simply velocity of B). The vectors ab and ob intersect at b .

3. Divide vector ab at c in the same ratio as C divides AB in the space diagram. In other words, $BC/CA = bc/ca$

4. Now from point c , draw vector cd perpendicular to CD to represent the velocity of D with respect to C (i.e. v_{DC}) and from point o draw vector od parallel to the path of motion of D (which along the vertical direction) to represent the velocity of D.

By measurement, we find that velocity of sliding at B,

$V_B = \text{vector } ob = 0.4$ m/s .

and velocity of sliding at D, $V_D = \text{vector } od = 0.24 \text{ m/s}$.

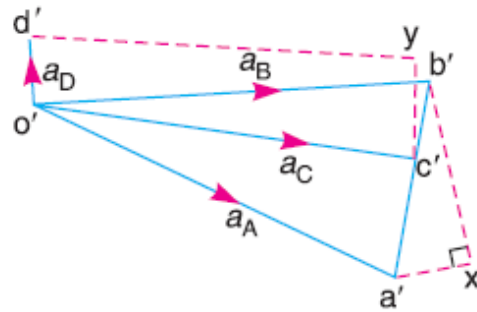
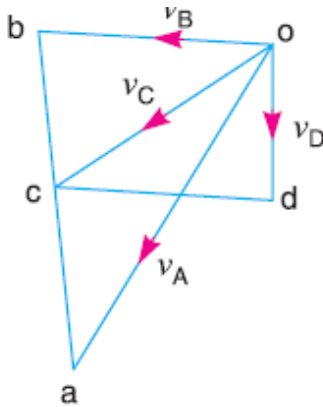
2. Angular velocity of CD

By measurement from velocity diagram, we find that velocity of D with respect to C,

$$V_{DC} = \text{vector } cd = 0.37 \text{ m/s}$$

\therefore Angular velocity of CD,

$$\omega_{CD} = V_{DC}/CD = 0.37/0.45 = 0.82 \text{ rad/s}$$



3. Linear acceleration of D

We know that the radial component of the acceleration of A with respect to O or acceleration of A,

$$a_{AO}^r = a_A = 1.231 \text{ m/s}^2$$

Radial component of the acceleration of B with respect to A,

$$a_{BA}^r = 0.243 \text{ m/s}^2$$

Radial component of the acceleration of D with respect to C,

$$a_{dc}^r = 0.304 \text{ m/s}^2$$

Now the acceleration diagram, as shown in Fig. 8.13 (c), is drawn as discussed below:

1. Draw vector $o'a'$ parallel to OA, to some suitable scale, to represent the radial component of the acceleration of A with respect to O or simply the acceleration of A, such that

$$\text{vector } o'a' = a_{OA}^r = a_A = 1.323 \text{ m/s}^2$$

2. From point a' , draw vector $a'x$ parallel to AB to represent the radial component of the acceleration of B with respect to A, such that

$$\text{vector } a'x = a_{BA}^r = 0.243 \text{ m/s}^2$$

3. From point x , draw vector xb' perpendicular to AB to represent the tangential component of the acceleration of B with respect to A (i.e. $BA a_{BA}^t$) whose magnitude is not yet known.

4. From point o' , draw vector $o'b'$ parallel to the path of motion of B (which is along BO) to represent the acceleration of B (a_B). The vectors xb' and $o'b'$ intersect at b' . Join $a'b'$. The vector $a'b'$ represents the acceleration of B with respect to A.

5. Divide vector $a'b'$ at c' in the same ratio as C divides AB in the space diagram. In other words, $BC / BA = b'c' / b'a'$

6. From point c' , draw vector $c'y$ parallel to CD to represent the radial component of the acceleration of D with respect to C, such that

$$\text{vector } c'y = a_{DC}^r = 0.304 \text{ m/s}^2$$

7. From point y , draw yd' perpendicular to CD to represent the tangential component of

acceleration of D with respect to C (i.e. a_{DC}^t) whose magnitude is not yet known.

8. From point o' , draw vector $o' d'$ parallel to the path of motion of D (which is along the vertical direction) to represent the acceleration of D (a_D). The vectors yd' and $o' d'$ intersect at d' .
By measurement, we find that linear acceleration of D,

$$a_D = \text{vector } o' d' = 0.16 \text{ m/s}^2 \text{ Ans.}$$

4. Angular acceleration of CD

From the acceleration diagram, we find that the tangential component of the acceleration of D with respect to C,

$$a_{DC}^t = yd' = 1.28 \text{ m/s}^2 \dots (\text{By measurement})$$

\therefore Angular acceleration of CD,

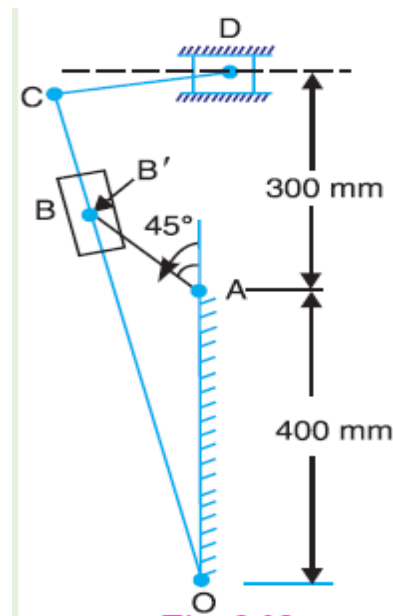
$$\alpha_{CD} = 2.84 \text{ rad/s}^2 \text{ (Clockwise)}$$

Example 2. A mechanism of a crank and slotted lever quick return motion is shown in Fig. If the crank rotates counter clockwise at 120 r.p.m., determine for the configuration shown, the velocity and acceleration of the ram D. Also determine the angular acceleration of the slotted lever. Crank, $AB = 150 \text{ mm}$; Slotted arm, $OC = 700 \text{ mm}$ and link $CD = 200 \text{ mm}$.

Solution. Given : $N_{BA} = 120 \text{ r.p.m}$ or $\omega_{BA} = 2\pi \times 120/60 = 12.57 \text{ rad/s}$; $AB = 150 \text{ mm} = 0.15 \text{ m}$; $OC = 700 \text{ mm} = 0.7 \text{ m}$; $CD = 200 \text{ mm} = 0.2 \text{ m}$

We know that velocity of B with respect to A,

$$V_{BA} = \omega_{BA} * AB = 1.9 \text{ m/s} \quad \dots (\text{Perpendicular to } AB)$$



Velocity of the ram D

First of all draw the space diagram, to some suitable scale, as shown in Fig. Now the velocity diagram, as shown in Fig. is drawn as discussed below:

1. Since O and A are fixed points, therefore these points are marked as one point in velocity diagram. Now draw vector ab in a direction perpendicular to AB , to some suitable scale, to represent the velocity of slider B with respect to A i.e. V_{BA} , such that

$$\text{vector } ab = V_{BA} = 1.9 \text{ m/s}$$

2. From point o, draw vector ob' perpendicular to OB' to represent the velocity of coincident point B' (on the link OC) with respect to O i.e. $v_{B'O}$ and from point b draw vector bb' parallel to the path of motion of B' (which is along the link OC) to represent the velocity of coincident point B' with respect to the slider B i.e. $v_{B'B}$. The vectors ob' and bb' intersect at b'.

3. Since the point C lies on OB' produced, therefore, divide vector ob' at c in the same ratio as C divides OB' in the space diagram. In other words, $ob' / oc = OB' / OC$

The vector oc represents the velocity of C with respect to O i.e. V_{CO} .

4. Now from point c, draw vector cd perpendicular to CD to represent the velocity of D with respect to C i.e. V_{DC} , and from point o draw vector od parallel to the path of motion of D (which is along the horizontal) to represent the velocity of D i.e. V_D . The vectors cd and od intersect at d. By measurement, we find that velocity of the ram D,

$$V_D = \text{vector } od = 2.15 \text{ m/s Ans.}$$

From velocity diagram, we also find that

Velocity of B with respect to B',

$$V_{BB'} = \text{vector } b'b = 1.05 \text{ m/s}$$

Velocity of D with respect to C,

$$V_{DC} = \text{vector } cd = 0.45 \text{ m/s}$$

Velocity of B' with respect to O

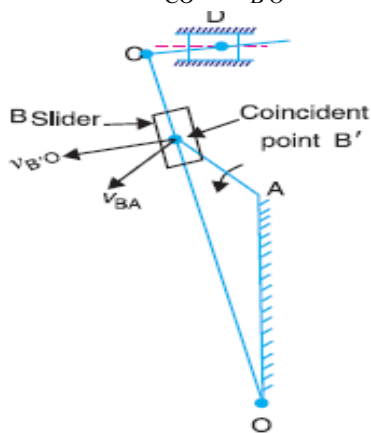
$$V_{B'O} = \text{vector } ob' = 1.55 \text{ m/s}$$

Velocity of C with respect to O,

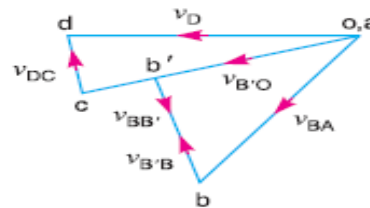
$$V_{CO} = \text{vector } oc = 2.15 \text{ m/s}$$

Angular velocity of the link OC or OB',

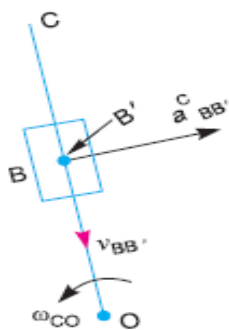
$$\omega_{CO} = \omega_{B'O} = 3.07 \text{ rad/s}$$



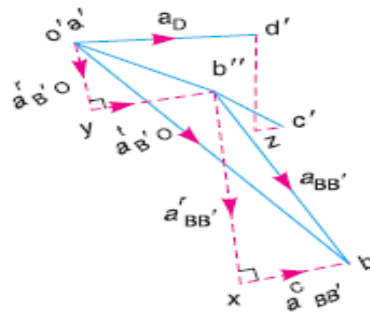
(a) Space diagram.



(b) Velocity diagram.



(c) Direction of coriolis component.



(d) Acceleration diagram.

Acceleration of the ram D

We know that radial component of the acceleration of B with respect to A,

$$a_{BA}^r = \omega_{CO} * AB = 23.7 \text{ m/s}^2$$

Coriolis component of the acceleration of slider B with respect to the coincident point B',

$$a_{BB'}^c = 2 \omega v = 6.45 \text{ m/s}^2 \quad \dots (\omega = \omega_{CO} \text{ and } v = v_{BB'})$$

Radial component of the acceleration of D with respect to C,

$$a_{DC}^r = v^2 DC / CD = 1.01 \text{ m/s}^2$$

Radial component of the acceleration of the coincident point B' with respect to O,

$$a_{B'O}^r = 4.62 \text{ m/s}^2 \dots (\text{By measurement } B'O = 0.52 \text{ m})$$

Now the acceleration diagram, as shown in Fig, is drawn as discussed below:

1. Since O and A are fixed points, therefore these points are marked as one point in the acceleration diagram. Draw vector a'b' parallel to AB, to some suitable scale, to represent the radial component of the acceleration of B with respect to A i.e. a_{BA}^r such that

$$\text{vector } a'b' = a_{BA}^r = 23.7 \text{ m/s}^2$$

2. The acceleration of the slider B with respect to the coincident point B' has the following two components :

(i) Coriolis component of the acceleration of B with respect to B' i.e. $a_{BB'}^c$, and

(ii) Radial component of the acceleration of B with respect to B' i.e. $a_{BB'}^r$.

These two components are mutually perpendicular. Therefore from point b' draw vector b'x perpendicular to B'O i.e. in a direction as shown in Fig. to represent

$a_{BB'}^c = 6.45 \text{ m/s}^2$. The direction of $a_{BB'}^c$ is obtained by rotating $V_{BB'}$ (represented by vector b'b in velocity diagram) through 90° in the same sense as that of link OC which rotates in the counter clockwise direction. Now from point x, draw vector xb'' perpendicular to vector b'x (or parallel to B'O) to represent $a_{BB'}^r$ whose magnitude is yet unknown.

3. The acceleration of the coincident point B' with respect to O has also the following two components:

(i) Radial component of the acceleration of coincident point B' with respect to O i.e.

$$a_{B'O}^r$$

(ii) Tangential component of the acceleration of coincident point B' with respect to O,

$$\text{i.e. } a_{B'O}^t.$$

These two components are mutually perpendicular. Therefore from point o', draw vector o'y parallel to B'O to represent $a_{B'O}^r = 4.62 \text{ m/s}^2$ and from point y draw vector yb'' perpendicular to vector o'y to represent $a_{B'O}^t$. The vectors xb'' and yb'' intersect at b''. Join o'b''. The vector o'b'' represents the acceleration of B' with respect to O, i.e. $a_{B'O}$.

4. Since the point C lies on OB' produced, therefore divide vector o'b'' at c' in the same ratio as C divides OB' in the space diagram. In other words,

$$o'b'' / o'c' = OB' / OC$$

5. The acceleration of the ram D with respect to C has also the following two components:

(i) Radial component of the acceleration of D with respect to C i.e. a_{DC}^r , and

(ii) Tangential component of the acceleration of D with respect to C, i.e. a_{DC}^t .

The two components are mutually perpendicular. Therefore draw vector c'z parallel to CD to represent $a_{DC}^r = 1.01 \text{ m/s}^2$ and from z draw zd' perpendicular to vector zc' to represent a_{DC}^t , at whose magnitude is yet unknown.

6. From point o', draw vector o'd' in the direction of motion of the ram D which is along the

horizontal. The vectors zd' and $o'd'$ intersect at d' . The vector $o'd'$ represents the acceleration of ram D i.e. a_D .

By measurement, we find that acceleration of the ram D,

$$a_D = \text{vector } o'd' = 8.4 \text{ m/s}^2 \text{ Ans.}$$

Angular acceleration of the slotted lever

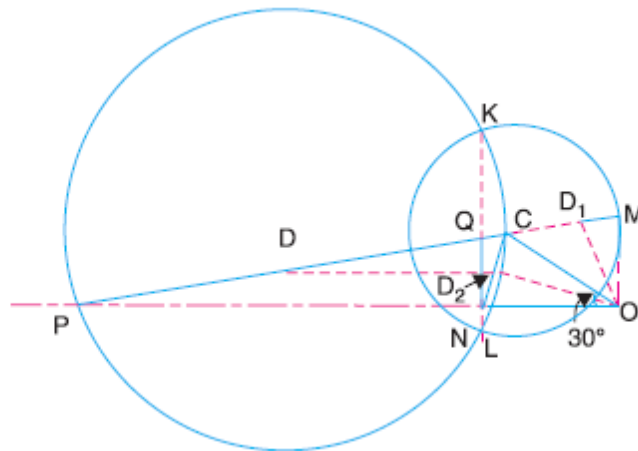
By measurement from acceleration diagram, we find that tangential component of the coincident point B' with respect to O, $a_{B'O}^t = \text{vector } yb'' = 6.4 \text{ m/s}^2$

We know that angular acceleration of the slotted lever,

$$a_{B'O}^t / OB' = 12.3 \text{ rad/s}^2$$

Example 4. The crank and connecting rod of a reciprocating engine are 200 mm and 700 mm respectively. The crank is rotating in clockwise direction at 120 rad/s. Find with the help of Klein's construction: 1. Velocity and acceleration of the piston, 2. Velocity and acceleration of the mid point of the connecting rod, and 3. Angular velocity and angular acceleration of the connecting rod, at the instant when the crank is at 30° to I.D.C. (inner dead centre).

Solution. Given: $OC = 200 \text{ mm} = 0.2 \text{ m}$; $PC = 700 \text{ mm} = 0.7 \text{ m}$; $\omega = 120 \text{ rad/s}$



The Klein's velocity diagram OCM and Klein's acceleration diagram CQNO as shown in Fig. is drawn to some suitable scale. By measurement, we find that

$OM = 127 \text{ mm} = 0.127 \text{ m}$; $CM = 173 \text{ mm} = 0.173 \text{ m}$; $QN = 93 \text{ mm} = 0.093 \text{ m}$; $NO = 200 \text{ mm} = 0.2 \text{ m}$

1. Velocity and acceleration of the piston

We know that the velocity of the piston P,

$$v_P = \omega \times OM = 120 \times 0.127 = 15.24 \text{ m/s}$$

and acceleration of the piston P,

$$a_P = \omega^2 \times NO = (120)^2 \times 0.2 = 2880 \text{ m/s}^2$$

2. Velocity and acceleration of the mid-point of the connecting rod

In order to find the velocity of the mid-point D of the connecting rod, divide CM at D_1 in the same ratio as D divides CP. Since D is the mid-point of CP, therefore D_1 is the mid-point of CM, i.e. $CD_1 = D_1M$. Join OD_1 . By measurement

$$OD_1 = 140 \text{ mm} = 0.14 \text{ m}$$

$$\text{Velocity of D, } v_D = \omega \times OD_1 = 120 \times 0.14 = 16.8 \text{ m/s}$$

In order to find the acceleration of the mid-point of the connecting rod, draw a line DD_2 parallel to the line of stroke PO which intersects CN at D_2 . By measurement

$$OD_2 = 193 \text{ mm} = 0.193 \text{ m}$$

Acceleration of D ,

$$a_D = \omega^2 \times OD_2 = (120)^2 \times 0.193 = 2779.2 \text{ m/s}^2$$

3. Angular velocity and angular acceleration of the connecting rod

We know that the velocity of the connecting rod PC (i.e. velocity of P with respect to C)

$$V_{PC} = \omega \times CM = 120 \times 0.173 = 20.76 \text{ m/s}$$

\therefore Angular acceleration of the connecting rod PC ,

$$\omega_{PC} = \frac{v_{PC}}{PC} = \frac{20.76}{0.7} = 29.66 \text{ rad/s Ans.}$$

We know that the tangential component of the acceleration of P with respect to C ,

$$a_{PC}^t = \omega^2 \times QN = (120)^2 \times 0.093 = 1339.2 \text{ m/s}^2$$

\therefore Angular acceleration of the connecting rod PC ,

$$\alpha_{PC} = \frac{a_{PC}^t}{PC} = \frac{1339.2}{0.7} = 1913.14 \text{ rad/s}^2 \text{ Ans.}$$

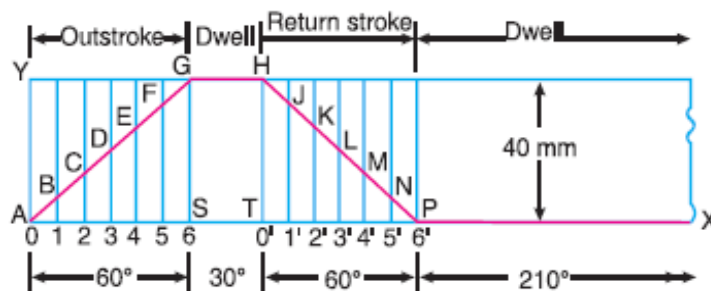
Experiment No-4

Solution of minimum three (including Graphical and Analytical) problem on cams

Example 1. A cam is to give the following motion to a knife-edged follower :1. Outstroke during 60° of cam rotation ; 2. Dwell for the next 30° of cam rotation ; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with uniform velocity during both the outstroke and return strokes. Draw the profile of the cam when (a) the axis of the follower passes through the axis of the cam shaft, and (b) the axis of the follower is offset by 20 mm from the axis of the cam shaft.

Solution following steps :

1. Draw a horizontal line $AX = 360^\circ$ to some suitable scale. On this line, mark $AS = 60^\circ$ to represent outstroke of the follower, $ST = 30^\circ$ to represent dwell, $TP = 60^\circ$ to represent return stroke and $PX = 210^\circ$ to represent dwell.
2. Draw vertical line AY equal to the stroke of the follower (i.e. 40 mm) and complete the rectangle as shown in Fig. 20.10.
3. Divide the angular displacement during outstroke and return stroke into any equal number of even parts (say six) and draw vertical lines through each point.
4. Since the follower moves with uniform velocity during outstroke and return stroke, therefore the displacement diagram consists of straight lines. Join AG and HP .
5. The complete displacement diagram is shown by $AGHPX$.



(a) Profile of the cam when the axis of follower passes through the axis of cam shaft

The profile of the cam when the axis of the follower passes through the axis of the cam shaft, is drawn as discussed in the following steps

1. Draw a base circle with radius equal to the minimum radius of the cam (i.e. 50 mm) with O as centre.
2. Since the axis of the follower passes through the axis of the cam shaft, therefore mark trace point A , .
3. From OA , mark angle $AOS = 60^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent

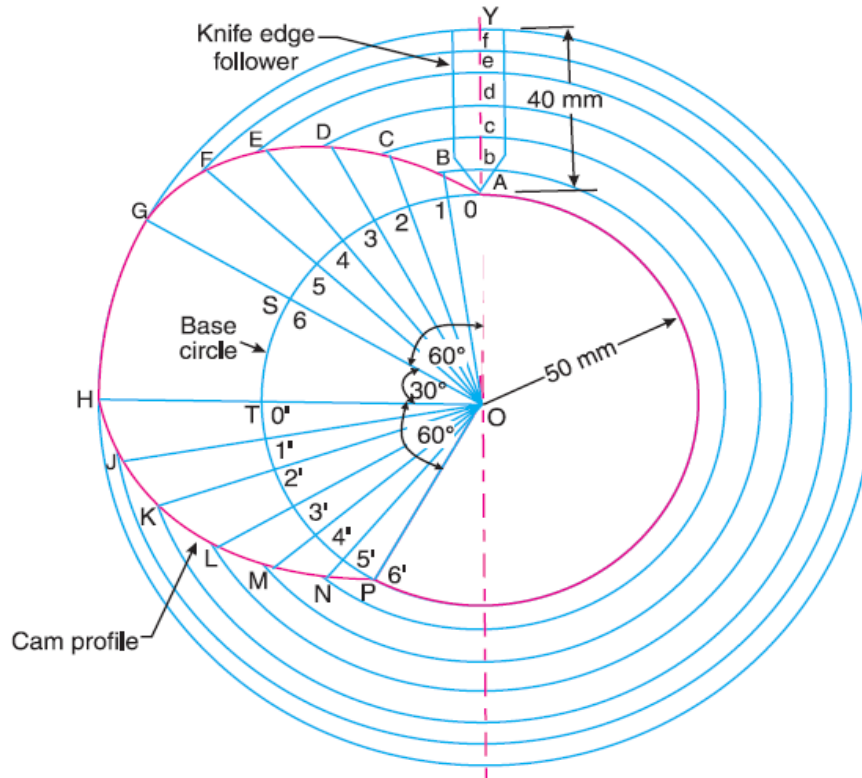
dwell and angle $TOP = 60^\circ$ to represent return stroke.

4. Divide the angular displacements during outstroke and return stroke (i.e. angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3 ...etc. and $0', 1', 2', 3', \dots$ etc. with centre O and produce beyond the base circle as shown in Fig. 20.11.

6. Now set off $1B, 2C, 3D \dots$ etc. and $0'H, 1'J \dots$ etc. from the displacement diagram.

7. Join the points A, B, C,... M, N, P with a smooth curve. The curve AGHPA is the complete profile of the cam.



(b) Profile of the cam when the axis of the follower is offset by 20 mm from the axis of the cam shaft

The profile of the cam when the axis of the follower is offset from the axis of the cam shaft, is drawn as discussed in the following steps :

1. Draw a base circle with radius equal to the minimum radius of the cam (i.e. 50 mm) with O as centre.

2. Draw the axis of the follower at a distance of 20 mm from the axis of the cam, which intersects the base circle at A.

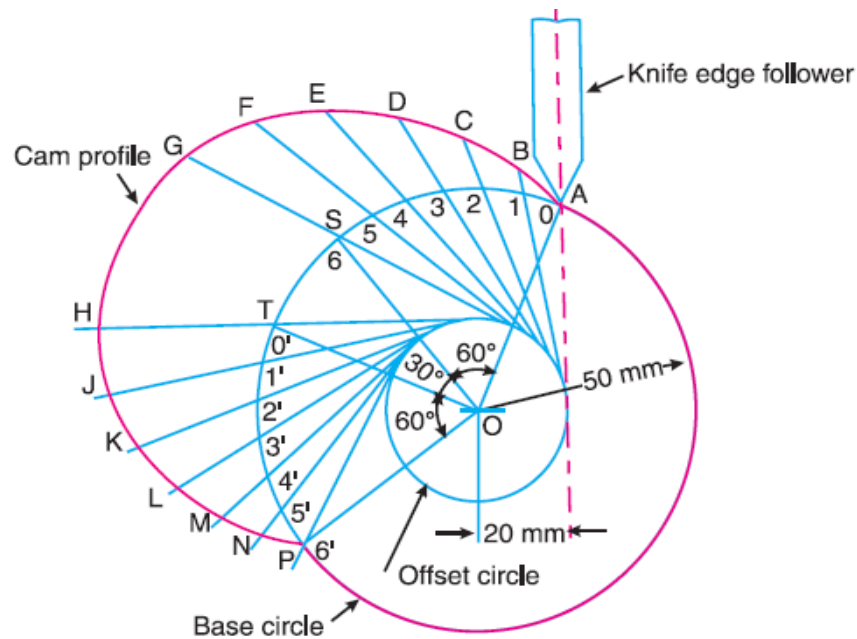
3. Join AO and draw an offset circle of radius 20 mm with centre O.

4. From OA, mark angle $AOS = 60^\circ$ to represent outstroke, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent return stroke.

5. Divide the angular displacement during outstroke and return stroke (i.e. angle AOS and angle TOP) into the same number of equal even parts as in displacement diagram.

6. Now from the points 1, 2, 3 ... etc. and $0', 1', 2', 3' \dots$ etc. on the base circle, draw tangents to the offset circle and produce these tangents beyond the base circle .

7. Now set off 1B, 2C, 3D ... etc. and 0' H, 1' J ... etc. from the displacement diagram.
8. Join the points A, B, C ...M, N, P with a smooth curve. The curve AGHPA is the complete profile of the cam.



Example 2. A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below :

1. To raise the valve through 50 mm during 120° rotation of the cam ;
2. To keep the valve fully raised through next 30° ;
3. To lower the valve during next 60° ; and
4. To keep the valve closed during rest of the revolution i.e. 150° ;

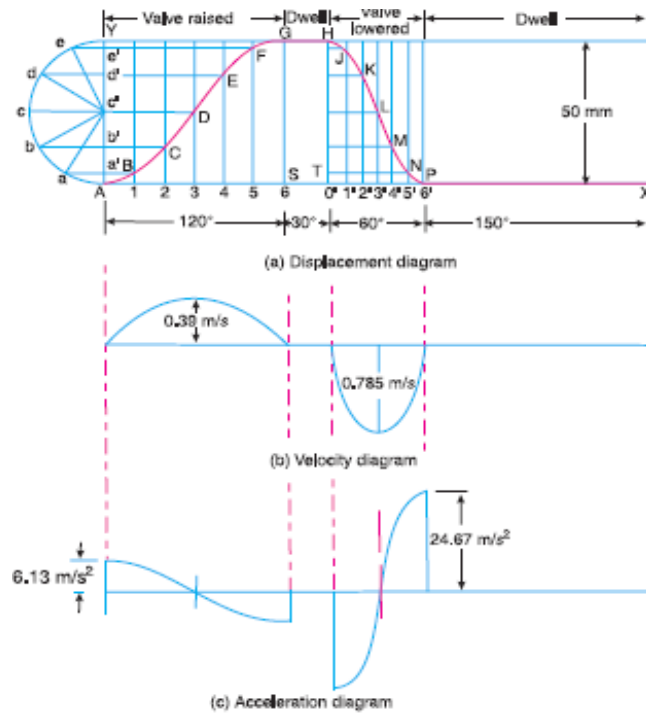
The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm.

Draw the profile of the cam when (a) the line of stroke of the valve rod passes through the axis of the cam shaft, and (b) the line of the stroke is offset 15 mm from the axis of the cam shaft. The displacement of the valve, while being raised and lowered, is to take place with simple harmonic motion. Determine the maximum acceleration of the valve rod when the cam shaft rotates at 100 r.p.m. Draw the displacement, the velocity and the acceleration diagrams for one complete revolution of the cam

Solution: Given : $S = 50 \text{ mm} = 0.05 \text{ m}$; $\theta O = 120^\circ = 2 \pi / 3 \text{ rad} = 2.1 \text{ rad}$;

$\theta R = 60^\circ = \pi / 3 \text{ rad} = 1.047 \text{ rad}$; $N = 100 \text{ r.p.m.}$

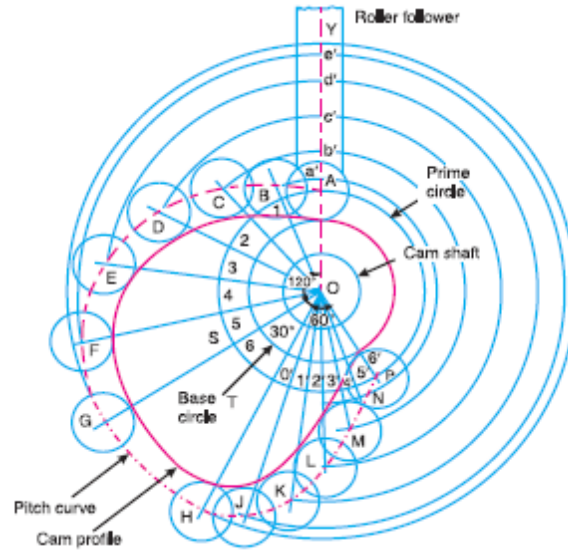
Since the valve is being raised and lowered with simple harmonic motion, therefore the displacement diagram, as shown in Fig.



(a) Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft

The profile of the cam, as shown in Fig. 20.17, is drawn as discussed in the following steps :

1. Draw a base circle with centre O and radius equal to the minimum radius of the cam (i.e. 25 mm).



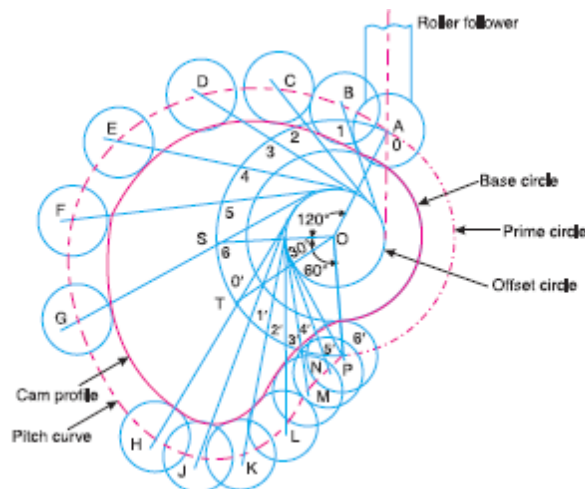
2. Draw a prime circle with centre O and radius,
 $OA = \text{Min. radius of cam} + 1/2 \text{ Dia. of roller} = 25 + 1/2 * 20 = 35 \text{ mm}$
3. Draw angle $AOS = 120^\circ$ to represent raising or out stroke of the valve, angle $SOT = 30^\circ$ to represent dwell and angle $TOP = 60^\circ$ to represent lowering or return stroke of the valve.
4. Divide the angular displacements of the cam during raising and lowering of the valve (i.e. angle AOS and TOP) into the same number of equal even parts as in displacement diagram.

5. Join the points 1, 2, 3, etc. with the centre O and produce the lines beyond prime circle as shown in Fig. 20.17.
6. Set off 1B, 2C, 3D etc. equal to the displacements from displacement diagram.
7. Join the points A, B, C ... N, P, A. The curve drawn through these points is known as pitch curve.
8. From the points A, B, C ... N, P, draw circles of radius equal to the radius of the roller.
9. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.17. This is the required profile of the cam.

(b) Profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft

The profile of the cam when the line of stroke is offset from the axis of the cam shaft, as shown in Fig. 20.18, may be drawn as discussed in the following steps :

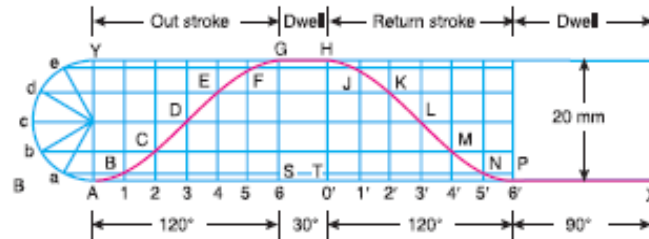
1. Draw a base circle with centre O and radius equal to 25 mm.
2. Draw a prime circle with centre O and radius OA = 35 mm.
3. Draw an off-set circle with centre O and radius equal to 15 mm.
4. Join OA. From OA draw the angular displacements of cam i.e. draw angle AOS = 120° , angle SOT = 30° and angle TOP = 60° .
5. Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (i.e. six parts) as in displacement diagram.
6. From points 1, 2, 3 etc. and 0', 1', 3', ...etc. on the prime circle, draw tangents to the offset circle.
7. Set off 1B, 2C, 3D... etc. equal to displacements as measured from displacement diagram.
8. By joining the points A, B, C ... M, N, P, with a smooth curve, we get a pitch curve.
9. Now A, B, C...etc. as centre, draw circles with radius equal to the radius of roller.
10. Join the bottoms of the circles with a smooth curve as shown in Fig. 20.18. This is the required profile of the cam.



Example 3. A cam drives a flat reciprocating follower in the following manner :During first 120° rotation of the cam, follower moves outwards through a distance of 20 mm with simple harmonic motion. The follower dwells during next 30° of cam rotation. During next 120° of cam rotation, the follower moves inwards with simple harmonic motion. The follower dwells for the next 90° of cam rotation. The minimum radius of the cam is 25 mm. Draw the profile of the cam.

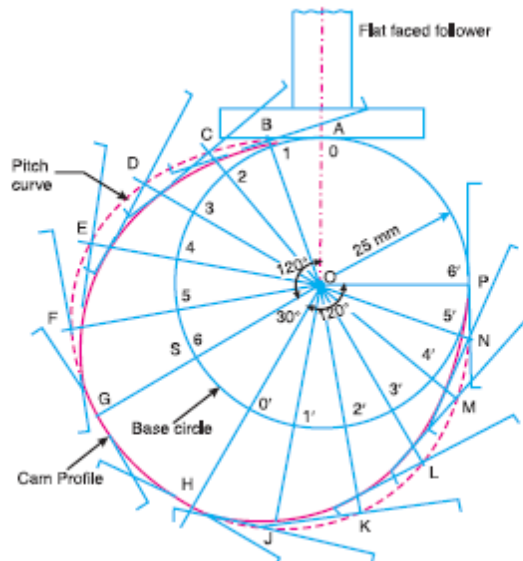
Construction

Since the follower moves outwards and inwards with simple harmonic motion, therefore the displacement diagram, as shown in fig.



Now the profile of the cam driving a flat reciprocating follower, as discussed in the following steps :

1. Draw a base circle with centre O and radius OA equal to the minimum radius of the cam (i.e. 25 mm).
2. Draw angle AOS = 120° to represent the outward stroke, angle SOT = 30° to represent dwell and angle TOP = 120° to represent inward stroke.



3. Divide the angular displacement during outward stroke and inward stroke (i.e. angles AOS and TOP) into the same number of equal even parts as in the displacement diagram.
5. From points 1, 2, 3 . . . etc., set off 1B, 2C, 3D . . . etc. equal to the distances measured from the displacement diagram.
6. Now at points B, C, D . . . M, N, P, draw the position of the flat-faced follower. The axis of the follower at all these positions passes through the cam centre.
7. The curve drawn tangentially to the flat side of the follower is the required profile of the Cam

Experiment:5

Solution of three problems on balancing of rotating masses

Example 1. A, B, C and D are four masses carried by a rotating shaft at radii 100, 125, 200 and 150 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the mass of B, C and D are 10 kg, 5 kg, and 4 kg respectively. Find the required mass A and the relative angular settings of the four masses so that the shaft shall be in complete balance.

Solution. Given : $r_A = 100 \text{ mm} = 0.1 \text{ m}$; $r_B = 125 \text{ mm} = 0.125 \text{ m}$; $r_C = 200 \text{ mm} = 0.2 \text{ m}$;
 $r_D = 150 \text{ mm} = 0.15 \text{ m}$; $m_B = 10 \text{ kg}$; $m_C = 5 \text{ kg}$; $m_D = 4 \text{ kg}$

The position of planes is shown in Fig. (a). Assuming the plane of mass A as the reference plane (R.P.), the data may be tabulated as below :

Plane	Mass(m)Kg	Radius (r)m	C.F./ ω^2 Kg.m	Distance from Plane A(l)m	Couple/ ω^2 Kg.m ²
A(R.P.)	m_A	0.1	$0.1 m_A$	0	0
B	10	0.125	1.25	0.6	0.75
C	5	0.2	0.2	1.2	1.2
D	4	0.15	0.15	1.8	1.08

First of all, the angular setting of masses C and D is obtained by drawing the couple polygon from the data given in Table 21.4 (column 6). Assume the position of mass B in the horizontal direction OB as shown in Fig. 21.10 (b). Now the couple polygon as shown in Fig. (c) is drawn as discussed below :

1. Draw vector $o' b'$ in the horizontal direction (i.e. parallel to OB) and equal to 0.75 kg-m^2 , to some suitable scale.
2. From points o' and b' , draw vectors $o' c'$ and $b' c'$ equal to 1.2 kg-m^2 and 1.08 kg-m^2 respectively. These vectors intersect at c' .
3. Now in Fig (b), draw OC parallel to vector $o' c'$ and OD parallel to vector $b' c'$. By measurement, we find that the angular setting of mass C from mass B in the anticlockwise direction, i.e. $\angle BOC = 240^\circ$.

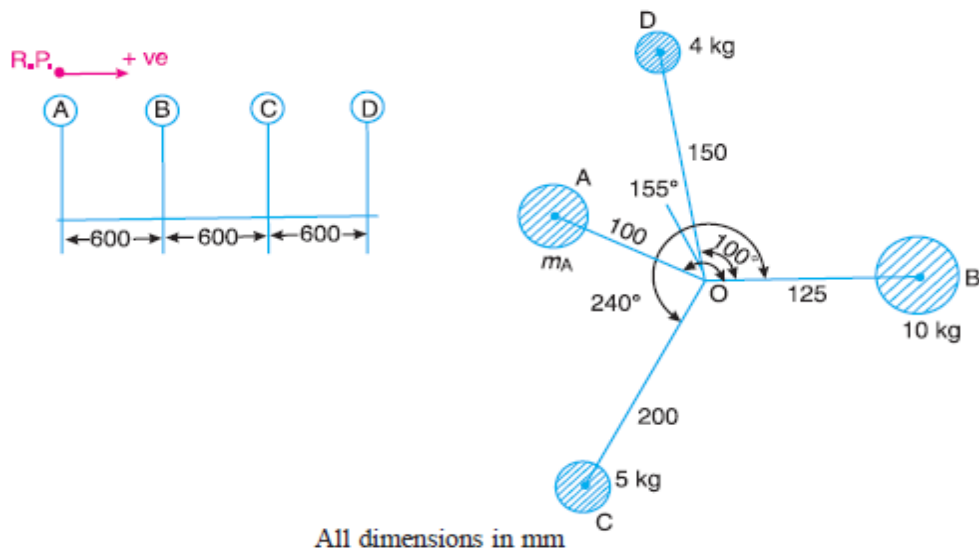
and angular setting of mass D from mass B in the anticlockwise direction, i.e. $\angle BOD = 100^\circ$

In order to find the required mass A (m_A) and its angular setting, draw the force polygon to some suitable scale, as shown in Fig.(d), from the data given in Table (column 4).

Since the closing side of the force polygon (vector do) is proportional to $0.1 m_A$, therefore by measurement,

$$0.1 m_A = 0.7 \text{ kg-m}^2 \text{ or } m_A = 7 \text{ kg}$$

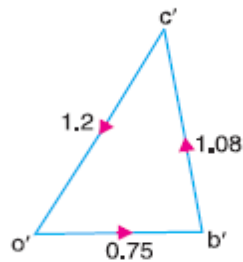
Now draw OA in Fig. 21.10 (b), parallel to vector do. By measurement, we find that the angular setting of mass A from mass B in the anticlockwise direction, i.e. $\angle BOA = 155^\circ$.



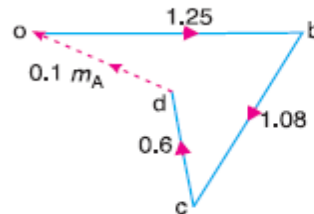
All dimensions in mm

(a) Position of planes.

(b) Angular position of masses.



(c) Couple polygon.



(d) Force polygon.

Example 2. A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Solution. Given : $m_A = 200$ kg ; $m_B = 300$ kg ; $m_C = 400$ kg ; $m_D = 200$ kg ; $r_A = 80$ mm = 0.08 m ; $r_B = 70$ mm = 0.07 m ; $r_C = 60$ mm = 0.06 m ; $r_D = 80$ mm = 0.08 m ; $r_X = r_Y = 100$ mm = 0.1 m

Let m_X = Balancing mass placed in plane X, and

m_Y = Balancing mass placed in plane Y.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. (a) and (b) respectively.

Assume the plane X as the reference plane (R.P.). The distances of the planes to the right of plane X are taken as + ve while the distances of the planes to the left of plane X are taken as - ve. The data may be tabulated as shown in Table.

Plane	Mass(m)Kg	Radius (r)m	C.F./ ω^2 Kg.m	Distance from Plane X(l)m	Couple/ ω^2 Kg.m ²
A	200	0.08	16	-0.1	-1.6
X(R.P.)	m_X	0.1	$0.1m_X$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2

Y	m_Y	0.1	$0.1m_Y$	0.4	$0.04m_Y$
D	200	0.08	16	0.6	9.6

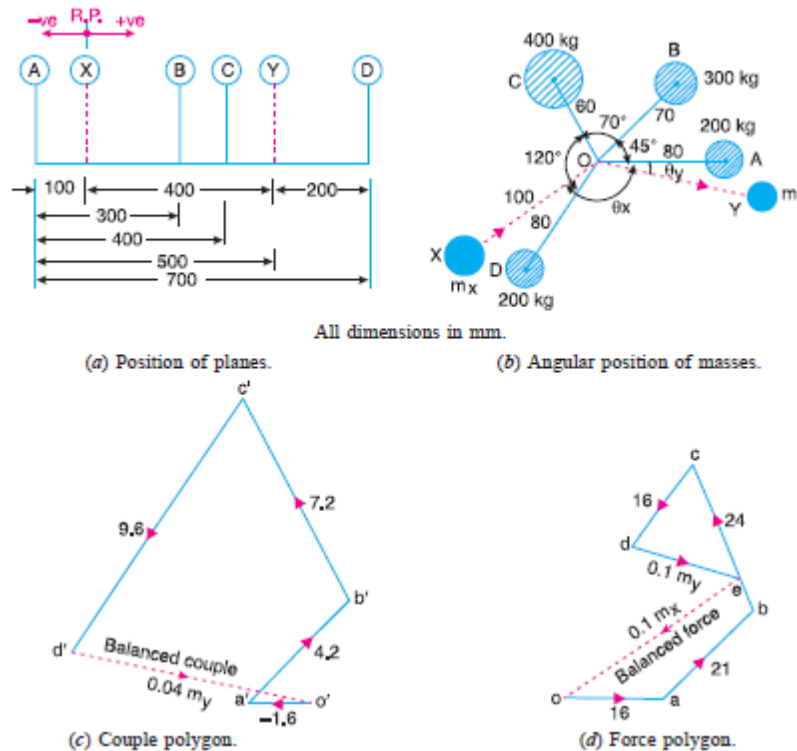
The balancing masses m_X and m_Y and their angular positions may be determined graphically as discussed below :

1. First of all, draw the couple polygon from the data given in Table 21.2 (column 6) as shown in Fig. 21.8 (c) to some suitable scale. The vector $d' o'$ represents the balanced couple. Since the balanced couple is proportional to $0.04 m_Y$, therefore by measurement, $0.04 m_Y = \text{vector } d' o' = 7.3 \text{ kg.m}^2$ or $m_Y = 182.5 \text{ kg}$

The angular position of the mass m_Y is obtained by drawing Om_Y in Fig (b), parallel to vector $d' o'$. By measurement, the angular position of m_Y is $\theta_Y = 12^\circ$ in the clockwise direction from mass m_A (i.e. 200 kg).

2. Now draw the force polygon from the data given in Table 21.2 (column 4) as shown in Fig. (d). The vector eo represents the balanced force. Since the balanced force is proportional to $0.1 m_X$, therefore by measurement, $0.1 m_X = \text{vector } eo = 35.5 \text{ kg-m}$ or $m_X = 355 \text{ kg}$.

The angular position of the mass m_X is obtained by drawing Om_X in Fig (b), parallel to vector eo . By measurement, the angular position of m_X is $\theta_X = 145^\circ$ in the clockwise direction from mass m_A (i.e. 200 kg).



Example 3. A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine :

1. relative angular positions of the pulleys, and
2. dynamic forces produced on the bearings when

the shaft rotates at 300 r.p.m.

Solution. Given : $m_A = 48 \text{ kg}$; $m_C = 20 \text{ kg}$; $r_A = 15 \text{ mm} = 0.015 \text{ m}$; $r_C = 12.5 \text{ mm} = 0.0125 \text{ m}$;
 $m_B = 56 \text{ kg}$; $r_B = 15 \text{ mm} = 0.015 \text{ m}$; $N = 300 \text{ r.p.m.}$ or $\omega = 2 \pi \times 300/60 = 31.42 \text{ rad/s}$

1. Relative angular position of the pulleys

The position of the shaft and pulleys is shown in Fig. (a).

Let m_L and $m_M =$ Mass at the bearings L and M, and

r_L and $r_M =$ Radius of rotation of the masses at L and M respectively.

Assuming the plane of bearing L as reference plane, the data may be tabulated as below

Plane	Mass(m)Kg	Radius (r)m	C.F./ ω^2 Kg.m	Distance from Plane L(l)m	Couple/ ω^2 Kg.m ²
A	48	0.015	0.72	-0.45	-0.324
L(R.P.)	m_L	r_L	$m_L r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M r_M$	1.8	$1.8 m_M r_M$
C	20	0.0125	0.25	2.25	0.5625

First of all, draw the force polygon to some suitable scale, as shown in Fig. 21.13 (c), from the data given in Table 21.7 (column 4). It is assumed that the mass of pulley B acts in vertical direction. We know that for the static balance of the pulleys, the centre of gravity of the system must lie on the axis of rotation. Therefore a force polygon must be a closed figure. Now in Fig.(b), draw OA parallel to vector bc and OC parallel to vector co. By measurement, we find that Angle between pulleys B and A = 161° Ans.

Angle between pulleys A and C = 76° Ans.

and Angle between pulleys C and B = 123°

2. Dynamic forces at the two bearings

In order to find the dynamic forces (or reactions) at the two bearings L and M, let us first calculate the values of $m_L.r_L$ and $m_M.r_M$ as discussed below :

1. Draw the couple polygon to some suitable scale, as shown in Fig.(d), from the data given in Table (column 6). The closing side of the polygon (vector $c' o'$) represents the balanced couple and is proportional to $1.8 m_M.r_M$. By measurement, we find that $1.8 m_M.r_M = \text{vector } c' o' = 0.97 \text{ kg-m}^2$ or $m_M.r_M = 0.54 \text{ kg-m}$

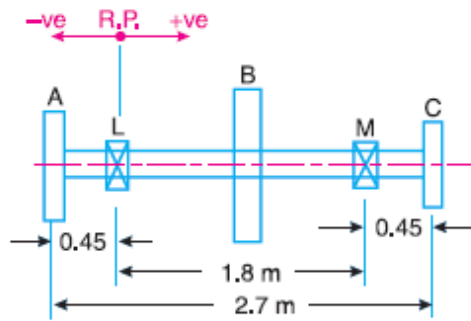
\therefore Dynamic force at the bearing M

$$m_M r_M \omega^2 = 0.54 (31.42)^2 = 533 \text{ N.}$$

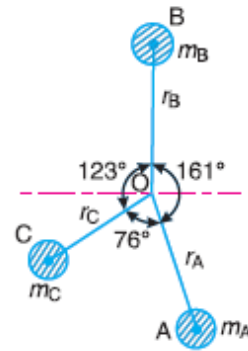
2. Now draw the force polygon, as shown in Fig.(e), from the data given in Table (column 4) and taking $m_M.r_M = 0.54 \text{ kg-m}$. The closing side of the polygon (vector do) represents the balanced force and is proportional to $m_L.r_L$. By measurement, we find that

$$m_L.r_L = 0.54 \text{ kg-m}$$

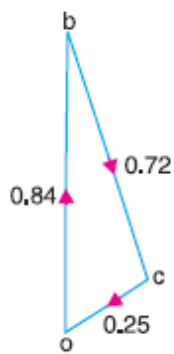
\therefore Dynamic force at the bearing L $m_L r_L \omega^2 = 0.54 (31.42)^2 = 533 \text{ N}$



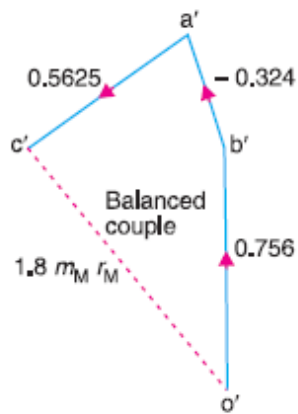
(a) Position of shaft and pulleys.



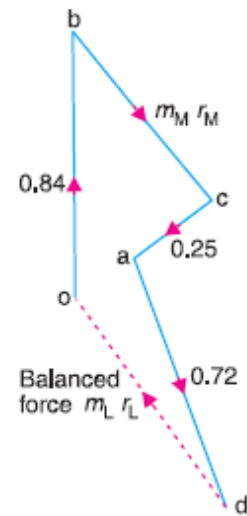
(b) Angular position of pulleys.



(c) Force polygon.



(d) Couple polygon.



(e) Force polygon.

Experiments No: 6

STUDY OF BRAKES

Introduction:

A **brake** is a device, by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air (or water which is circulated through the passages in the brake drum) so that excessive heating of the brake lining does not take place. The capacity of a brake depends upon the following factors:

1. The unit pressure between the braking surfaces,
2. The coefficient of friction between the braking surfaces,
3. The peripheral velocity of the brake drum,
4. The projected area of the friction surfaces, and
5. The ability of the brake to dissipate heat equivalent to the energy being absorbed.

The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

Types of Brakes:

The brakes, according to the means used for transforming the energy by the braking elements, are classified as:

1. Hydraulic brakes e.g. pumps or hydrodynamic brake and fluid agitator,
2. Electric brakes e.g. generators and eddy current brakes, and
3. Mechanical brakes.

The hydraulic and electric brakes cannot bring the member to rest and are mostly used where large amounts of

energy are to be transformed while the brake is retarding the load such as in laboratory dynamometers, high way trucks and electric locomotives. These brakes are also used for retarding or controlling the speed of a vehicle for down-hill travel. The mechanical brakes, according to the direction of acting force, may be divided into the following two groups:

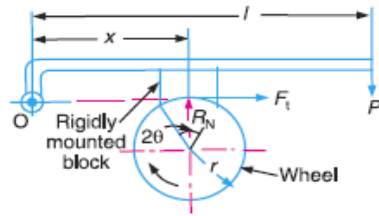
(a) Radial brakes. In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into external brakes and internal brakes. According to the shape of the friction elements, these brakes may be block or shoe brakes and band brakes.

(b) Axial brakes. In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes. The analysis of these brakes is similar to clutches. Since we are concerned with only mechanical brakes, therefore, these are discussed, in detail, in the following pages.

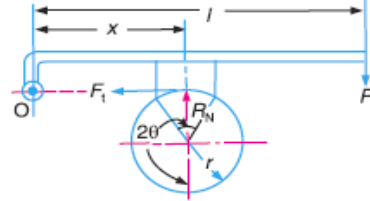
1. Single Block or Shoe Brake

A single block or shoe brake is shown in Fig... It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel,

which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. The other end of the lever is pivoted on a fixed fulcrum O.



(a) Clockwise rotation of brake wheel



(b) Anticlockwise rotation of brake wheel.

P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60° , then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu \cdot R_N$$

and the braking torque

$$T_B = F_t \cdot r = \mu \cdot R_N \cdot r$$

Let us now consider the following three cases:

Case 1.

When the line of action of tangential braking force (F_t) passes through the fulcrum O of the lever,

and the brake wheel rotates clockwise as shown in Fig. then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

Braking torque,

$$T_B = \mu \cdot R_N \cdot r = \mu \times \frac{P \cdot l}{x} \times r = \frac{\mu \cdot P \cdot l \cdot r}{x}$$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (b), then the braking torque is same, i.e.

$$T_B = \mu \cdot R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x}$$

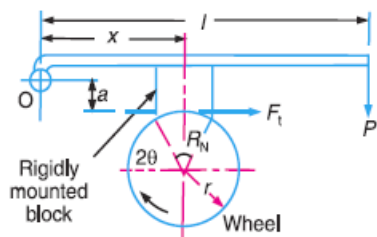
Case 2.

When the line of action of the tangential braking force (F_t) passes through a distance 'a' below the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. 19.2 (a), then for equilibrium, taking moments about the fulcrum O,

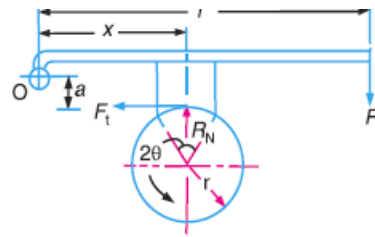
$$R_N \times x + F_t \times a = P \cdot l \text{ or } R_N \times x + \mu R_N \times a = P \cdot l \text{ or } R_N = \frac{P \cdot l}{x + \mu \cdot a}$$

and braking torque,

$$T_B = \mu R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x + \mu \cdot a}$$



(a) Clockwise rotation of brake wheel.



(b) Anticlockwise rotation of brake wheel.

When the brake wheel rotates anticlockwise, as shown in Fig. then for equilibrium,

$$R_N \cdot x = P \cdot l + F_t \cdot a = P \cdot l + \mu R_N \cdot a$$

$$\text{or } R_N (x - \mu \cdot a) = P \cdot l \quad \text{or } R_N = \frac{P \cdot l}{x - \mu \cdot a}$$

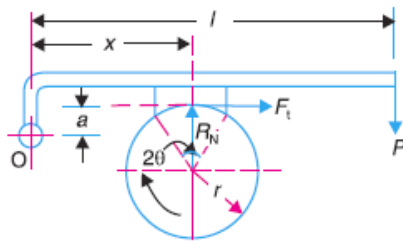
$$\text{and braking torque, } T_B = \mu R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x - \mu \cdot a}$$

Case 3.

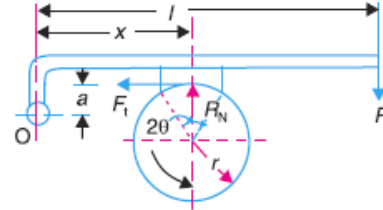
When the line of action of the tangential braking force (F_t) passes through a distance 'a' above the fulcrum O, and the brake wheel rotates clockwise as shown in Fig. then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \cdot x = P \cdot l + F_t \cdot a = P \cdot l + \mu R_N \cdot a$$

$$R_N (x - \mu \cdot a) = P \cdot l \quad \text{or } R_N = \frac{P \cdot l}{x - \mu \cdot a}$$



(a) Clockwise rotation of brake wheel.



(b) Anticlockwise rotation of brake wheel.

And braking torque,

$$T_B = \mu R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x - \mu \cdot a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (b), then for equilibrium, taking moments about the fulcrum O, we have

$$R_N \cdot x + F_t \cdot a = P \cdot l \quad \text{or } R_N \cdot x + \mu R_N \cdot a = P \cdot l$$

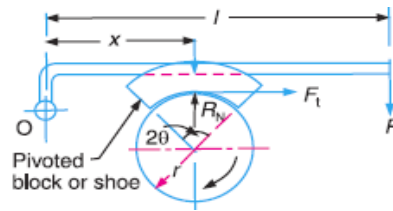
$$\text{and braking torque, } T_B = \mu R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x + \mu \cdot a} \quad \text{or } R_N = \frac{P \cdot l}{x + \mu \cdot a}$$

and braking torque,

$$T_B = \mu R_N \cdot r = \frac{\mu \cdot P \cdot l \cdot r}{x + \mu \cdot a}$$

2. Pivoted Block or Shoe Brake

We have discussed in the previous article that when the angle of contact is less than 60° , then when the angle of contact is greater than 60° , then the unit pressure normal to the surface of contact is less at the ends than at the centre. In such cases, the block or shoe is pivoted to the lever, as shown in Fig. 19.4, instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque for a pivoted block or shoe brake (i.e. when $2\theta > 60^\circ$) is given by



$$T_B = F_t \times r = \mu' \cdot R_N \cdot r$$

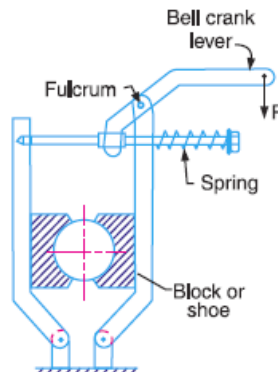
$$\mu' = \text{Equivalent coefficient of friction} = \frac{4\mu \sin \theta}{2\theta + \sin 2\theta}, \text{ and}$$

$$\mu = \text{Actual coefficient of friction.}$$

These brakes have more life and may provide a higher braking torque.

3. Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (RN). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid. When the current is switched off, there is no force on the bell crank lever and the brake is engaged automatically due to the spring force and thus there will be no downward movement of the load.



In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$$T_B = (F_{t1} + F_{t2}) r$$

Where F_{t1} and F_{t2} are the braking forces on the two blocks

3. Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig, is called a simple band brake in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum. When a force P is applied to the lever at C , the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum provides the braking force. The force P on the lever at C may be determined as discussed below:

T_1 = Tension in the tight side of the band,

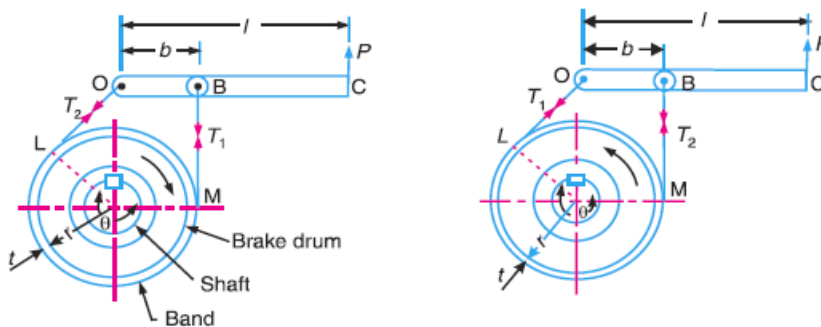
T_2 = Tension in the slack side of the band

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and



We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu\theta} \quad \text{or} \quad 2.3 \log \left(\frac{T_1}{T_2} \right) = \mu\theta$$

and braking force on the drum = $T_1 - T_2$

\therefore Braking torque on the drum,

$$T_B = (T_1 - T_2) r \quad \dots \text{(Neglecting thickness of band)}$$

$$= (T_1 - T_2) r_e \quad \dots \text{(Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1 . On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. the tensions in the band will reverse, i.e. the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2 . Now taking moments about the fulcrum O , we have

$$P.l = T_1.b$$

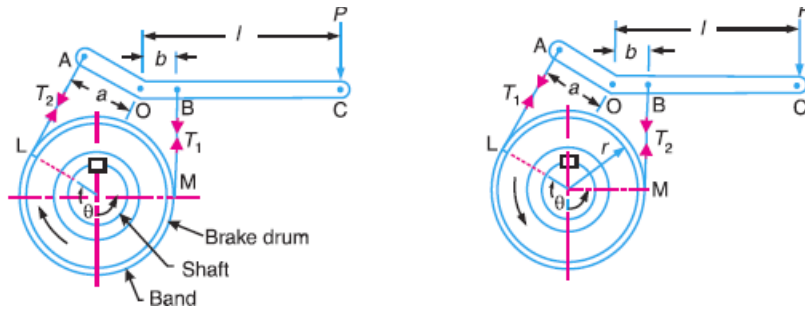
$$P.l = T_2.b$$

l = Length of the lever from the fulcrum (OC), and

b = Perpendicular distance from O to the line of action of T_1 or T_2 .

4. Differential Band Brake:

In a differential band brake, as shown in Fig., the ends of the band are joined at A and B to a lever AOC pivoted on a fixed pin or fulcrum O . It may be noted that for the band to tighten, the length OA must be greater than the length OB .



The braking torque on the drum may be obtained in the similar way as discussed in simple band brake. Now Considering the equilibrium of the lever AOC. It may be noted that when the drum rotates in the clockwise direction, as shown in Fig. , the end of the band attached to A will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1 . On the other hand, when the drum rotates in the anticlockwise direction, as shown in Fig. the end of the band attached to A will be tight with tension T_1 and end of the band attached to B will be slack with tension T_2 . Now taking moments about the fulcrum O, we have

$$P.l + T_1.b = T_2.a$$

... (For clockwise rotation of the drum)

$$P.l = T_2.a - T_1.b$$

$$P.l + T_2.b = T_1.a$$

... (For anticlockwise rotation of the drum)

$$P.l = T_1.a - T_2.b$$

We have discussed in block brakes that when the frictional force helps to apply the brake, it is said to be self energizing brake. In case of differential band brake, we see from equations (i) and (ii) that the moment $T_1.b$ and $T_2.b$ helps in applying the brake (because it adds to the moment $P.l$) for the clockwise and anticlockwise rotation of the drum respectively. We have also discussed that when the force P is negative or zero, then brake is self locking. Thus for differential band brake and for clockwise rotation of the drum, the condition for self locking

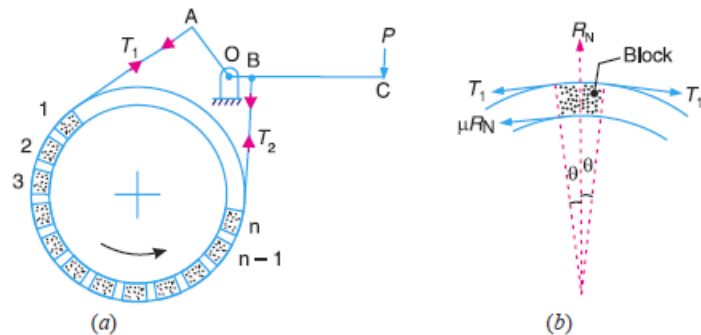
$$T_2.a < T_1.b \text{ or } T_2 / T_1 < b / a$$

and for anticlockwise rotation of the drum, the condition for self locking is

$$T_1.a < T_2.b \text{ or } T_1 / T_2 < b / a$$

5. Band and Block Brake

The band brake may be lined with blocks of wood or other material, as shown in Fig. The friction between the blocks and the drum provides braking action. Let there are 'n' number of blocks, each subtending an angle 2θ at the centre and the drum rotates in anticlockwise direction.



- T_1 = Tension in the tight side,
 T_2 = Tension in the slack side,
 μ = Coefficient of friction between the blocks and drum,
 T_1' = Tension in the band between the first and second block,
 T_2', T_3' etc. = Tensions in the band between the second and third block,
 between the third and fourth block etc.

Consider one of the blocks (say first block) as shown in Fig. This is in equilibrium under the action of the following forces:

1. Tension in the tight side (T_1),
2. Tension in the slack side (T_1') or tension in the band between the first and second block,
3. Normal reaction of the drum on the block (R_N), and
4. The force of friction ($\mu.R_N$).

Resolving the forces radially, we have

$$(T_1 - T_1') \sin \theta = R_N$$

Resolving the forces tangentially, we have

$$(T_1 + T_1') \cos \theta = \mu.R_N$$

Dividing equation (ii) by (i), we have

$$\frac{(T_1 - T_1') \cos \theta}{(T_1 + T_1') \sin \theta} = \frac{\mu.R_N}{R_N}$$

$$\text{or} \quad (T_1 - T_1') = \mu \tan \theta (T_1 + T_1')$$

$$\therefore \quad \frac{T_1}{T_1'} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

Similarly, it can be proved for each of the blocks that

$$\frac{T_1'}{T_2'} = \frac{T_2'}{T_3'} = \frac{T_3'}{T_4'} = \dots = \frac{T_{n-1}'}{T_n'} = \frac{1 + \mu \tan \theta}{1 - \mu \tan \theta}$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_1}{T_1'} \times \frac{T_1'}{T_2'} \times \frac{T_2'}{T_3'} \times \dots \times \frac{T_{n-1}'}{T_n'} = \left(\frac{1 + \mu \tan \theta}{1 - \mu \tan \theta} \right)^n \quad \dots (iii)$$

Braking torque on the drum of effective radius r_e ,

$$\begin{aligned}
 T_B &= (T_1 - T_2) r_e \\
 &= (T_1 - T_2) r \quad \dots \text{[Neglecting thickness of band]}
 \end{aligned}$$

Experiment No: 07

Study of Dynamometer

Introduction:

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers :

Following are the two types of dynamometers, used for measuring the brake power of an engine.

1. Absorption dynamometers, and 2. Transmission dynamometers.

In the absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the transmission dynamometers, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

Classification of Absorption Dynamometers:

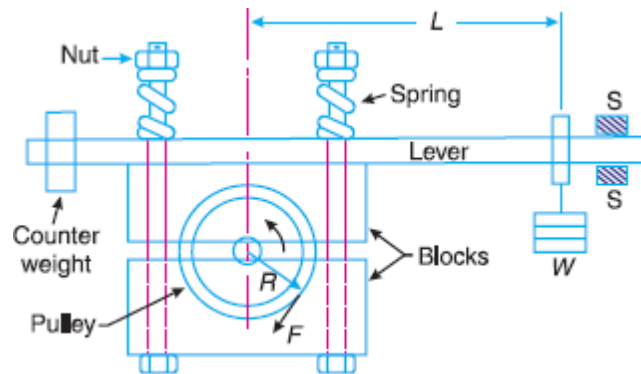
The following two types of absorption dynamometers are important from the subject point of view:

1. Prony brake dynamometer, and 2. Rope brake dynamometer.

These dynamometers are discussed, in detail, in the following pages.

1. Prony Brake Dynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig. A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight W at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight W must balance the moment of the frictional resistance between the blocks and the pulley.

W = Weight at the outer end of the lever in newtons,

L = Horizontal distance of the weight W from the centre of the pulley in metres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and N = Speed of the shaft in r.p.m.
 We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.L = F.R \text{ N-m}$$

$$\begin{aligned} \text{Work done in one revolution} &= \text{Torque} \times \text{Angle turned in radians} \\ &= T \times 2\pi \text{ N-m} \end{aligned}$$

$$\text{Work done per minute} = T \times 2\pi N \text{ N-m}$$

We know that brake power of the engine

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

2.Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel. In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

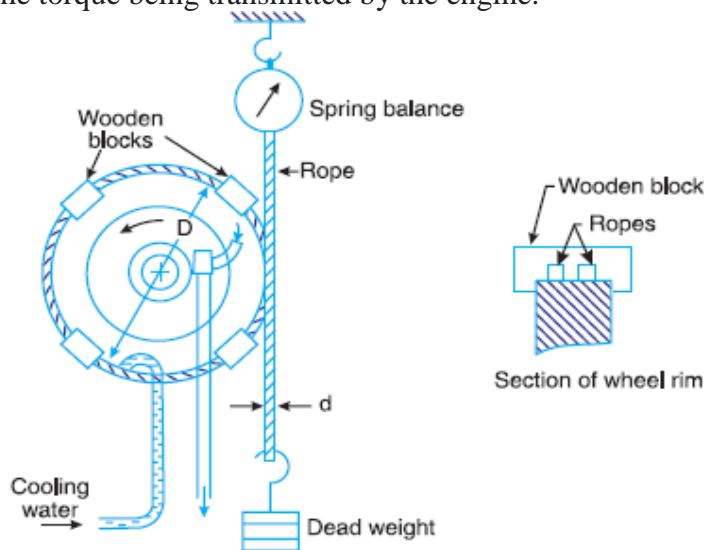


Fig. 19.32. Rope brake dynamometer.

∴ Brake power of the engine,

$$B.P = \frac{\text{Work done per min}}{60} = \frac{(W - S) \pi (D + d) N}{60} \text{ watts}$$

W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in metres,

d = diameter of rope in metres, and

N = Speed of the engine shaft in r.p.m.

Net load on the brake

$$= (W - S) N$$

We know that distance moved in one revolution

$$= \pi (D + d) \text{ m}$$

$$\begin{aligned} & \text{Work done per revolution} \\ & = (W - S) \pi (D + d) \text{ N-m} \\ & \text{and work done per minute} \\ & = (W - S) \pi (D + d) N \text{ N-m} \end{aligned}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$\text{B.P.} = \frac{(W - S) \pi D N}{60} \text{ watts}$$

Classification of Transmission Dynamometers:

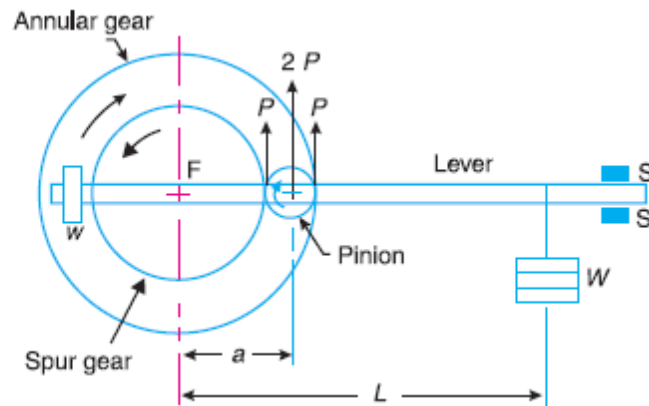
The following types of transmission dynamometers are important from the subject point of view :

1. Epicyclic-train dynamometer, 2. Belt transmission dynamometer, and 3. Torsion dynamometer.

We shall now discuss these dynamometers, in detail, in the following pages.

1. Epicyclic-train Dynamometer:

An epicyclic-train dynamometer, as shown in Fig., consists of a simple epicyclic train of gears, i.e. a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (i.e. driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight w is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the friction of the pin on which the pinion rotates is neglected, then the tangential effort P exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion are equal. Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is $2P$.



This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight W at the end of the lever. The stops S, S are provided to control the movement of the lever.

For equilibrium of the lever, taking moments about the fulcrum F ,

$$2P \times a = W.L \text{ or } P = W.L / 2a$$

Let R = Pitch circle radius of the spur gear in metres, and

N = Speed of the engine shaft in r.p.m.

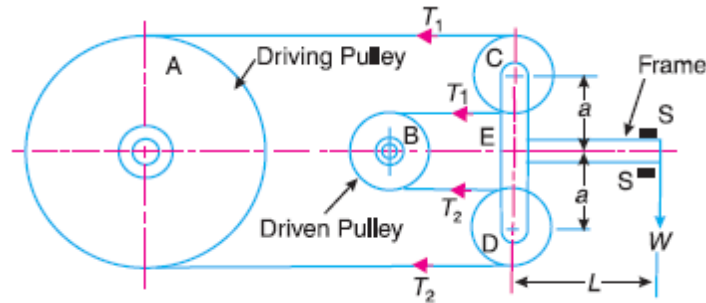
$$\text{Torque transmitted, } T = P.R$$

and power transmitted

$$= \frac{T \times 2\pi N}{60} = \frac{P.R \times 2\pi N}{60} \text{ watts}$$

2. Belt Transmission Dynamometer-Froude or Thorneycroft Transmission Dynamometer:

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it is running



A belt transmission dynamometer, as shown in Fig. is called a Froude or Thorneycroft transmission dynamometer. It consists of a pulley A (called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley B (called driven pulley) mounted on another shaft to which the power from pulley A is transmitted. The pulleys A and B are connected by means of a continuous belt passing round the two loose pulleys C and D which are mounted on a T-shaped frame. The frame is pivoted at E and its movement is controlled by two stops S,S. Since the tension in the tight side of the belt (T_1) is greater than the tension in the slack side of the belt (T_2), therefore the total force acting on the pulley C (i.e. $2T_1$) is greater than the total force acting on the pulley D (i.e. $2T_2$). It is thus obvious that the frame causes movement about E in the anticlockwise direction. In order to balance it, a weight W is applied at a distance L from E on the frame as shown in fig.

Now taking moments about the pivot E, neglecting friction,

$$2T_1 \times a = 2T_2 \times a + W.L \quad \text{or} \quad T_1 - T_2 = \frac{W.L}{2a}$$

Let D = diameter of the pulley A in metres, and
 N = Speed of the engine shaft in r.p.m.

$$\therefore \text{Work done in one revolution} = (T_1 - T_2) \pi D N \text{-m}$$

$$\text{and workdone per minute} = (T_1 - T_2) \pi D N N \text{-m}$$

$$\therefore \text{Brake power of the engine, B.P.} = \frac{(T_1 - T_2) \pi D N}{60} \text{ watts}$$

3. Torsion Dynamometer:

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (T), length of the shaft (l), diameter of the shaft (D) and modulus of rigidity (C) of the material of the shaft. We know that the torsion equation is

$$\frac{T}{J} = \frac{C\theta}{l}$$

where

θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter D , the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter D and internal diameter d , the polar moment of inertia,

$$J = \frac{\pi}{32} (D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{CJ}{l} \times \theta = k\theta$$

where $k = CJ/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined.

We know that the power transmitted

$$P = \frac{T \times 2\pi N}{60} \text{ watts, where } N \text{ is the speed in r.p.m.}$$

3. Quiz on the subject:-

I. Introduction and Definitions

1. What is kinematics and dynamics?
2. Define higher pair with one example.
3. What is successfully constrained motion?
4. Calculate degree of freedom of four bar mechanism with all turning pairs.
5. State Grashof's law for four bar mechanism.
6. State Kennedy's theorem of three instantaneous centres.
7. Sketch Withworth's Quick return mechanism.
8. Distinguish between mechanisms.
9. Differentiate between lower and higher pair giving example.
10. Sketch any two inversions of double slider crank mechanism.
11. What is Grubler's criterion for degree of freedom of mechanism?
12. What is kinetics and statics?
13. What are rigid and resistant bodies?
14. What is degree of freedom of mechanism?
15. Differentiate between closed and unclosed pair.
16. Define kinematic pair and kinematic chain.
17. Define Grashof's Law.
18. Sketch any two inversion of slider crank mechanism.
19. Differentiate between turning and rolling pair.

II. Velocity Analysis

1. In a mechanism the fixed instantaneous centres are those which
 - a) Remain in the same place for all configurations of the mechanism.
 - b) Vary with the configuration of the mechanism
 - c) Moves as the mechanism moves, but joints are of permanent nature
 - d) None of above.

2. When the slider moves on a fixed link having a curved surface, their instantaneous centre lies.

- a) On their point of contact
- b) At the centre of curvature
- c) At the pin point
- d) At the centre of circle

3. What is velocity image?

4. What is velocity of rubbing?

5. State Kennedy's theorem applicable to instantaneous centre of rotation.

6. Define body centrode and space centrode.

7. What is instantaneous centre of rotation of a link in mechanism?

III. Acceleration Analysis

1. What is acceleration image?

2. Formulate two components of acceleration.

3. The Coriolis component of acceleration is taken into account for

- a) Slider crank mechanism
- b) Four bar chain mechanism
- c) Quick return motion mechanism
- d) None of those

4. When a point at the end of a link moves with constant angular velocity, its acceleration will have

- a) Tangential components only
- b) Radial components only
- c) Coriolis component only
- d) Radial and tangential components both

5. The component of the acceleration parallel to velocity of the particle at the given instant is called

- a) Tangential component
- b) Radial component

c) Coriolis component

d) None of these

6. What are centripetal and tangential components of acceleration?

IV. Brake and dynamometer & CAMS

1. Sketch and label cam profiles.

2. Why radial follower is preferred to that of knife edge follower?

3. What is displacement diagram in cam?

4. Distinguish between brake and dynamometer.

5. Enumerate types of brakes.

6. Draw neat diagram of internal expanding brake.

7. Sketch epicyclic train dynamometer.

8. Classify cams according to motion of follower.

9. What is meant by self locking of brake?

10. Define base circle, pitch circle and pressure angle applied to cam.

11. Explain working of band and block brake.

12. What is advantage of self expanding brake?

IV. Balancing of Rotating Masses

1. Explain concept of balancing.

2. What do you mean by primary unbalancing?

3. What do you mean by static and dynamic unbalance?

4. What is necessity of balancing?

IV. Balancing of Reciprocating Masses

1. Deduce expression for swaying couple.

2. Deduce expression for hammer blow.

3. What do you mean by secondary unbalance in reciprocating engines?

4. The method of direct and reverse crank is used in engines for

a) The control of speed fluctuation

b)_Balancing of forces and couple

c)Kinematic analysis

d)Vibration analysis

5.In reciprocating engine primary forces

a)Are completely balanced

b)Are partially balanced

c)Are balanced by secondary forces

d)Cannot be balanced

4. Conduction of Viva-Voce Examinations:

Teacher should conduct oral exams of the students with full preparation. Normally, the objective questions with guess are to be avoided. To make it meaningful, the questions should be such that depth of the students in the subject is tested. Oral examinations are to be conducted in cordial environment amongst the teachers taking the examination. Teachers taking such examinations should not have ill thoughts about each other and courtesies should be offered to each other in case of difference of opinion, which should be critically suppressed in front of the students.

5. Evaluation and marking system:

Basic honesty in the evaluation and marking system is absolutely essential and in the process impartial nature of the evaluator is required in the examination system to become. It is a primary responsibility of the teacher to see that right students who are really putting up lot of hard work with right kind of intelligence are correctly awarded.

The marking patterns should be justifiable to the students without any ambiguity and teacher should see that students are faced with just circumstances.